

# Supplementary material document

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**Abstract.** We present the simulation results of Relativistic Runaway Electron Avalanches produced in air at sea level. The data presented here results from runs of three codes used in the High Energy Atmospheric Physics community :

- Geant4 (two set-ups)
- GRRR
- REAM

The results are extracted from two simulations :

- One evaluating the probability to produce a Relativistic Runaway Electron Avalanche (RREA) from a given seed electron inside a given electric field.
- One generating a RREA from seed electrons and recording produced particles at different time and distances, and extracting important parameters from it.

We also detail these two simulation set-ups in order for other researchers to be able to reproduce them.

Some of the presented plots go beyond the code comparison and are presented to support information given in the main article (see main article).

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## Note Concerning Geant4 Simulations

For the RREA probability simulations we set  $\alpha_R = 0.001$  (c.f. main text for more information). For the RREA characteristics simulations, we use the maximum step of 1 mm for the results presented here (see main text for more information).

## Medium composition

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- The air is composed of 78.085% nitrogen, 20.95% oxygen and 0.965% argon,
- Absolute number density of  $2.688 \cdot 10^{25} \text{ m}^{-3}$ , equivalent to  $1.293 \text{ kg.m}^{-3}$ . We always use an uniform air density.

## Coordinate system and domain

- Cartesian coordinates x, y, z.
- Any field is always applied in the z (or -z) direction.
- Domain is a cylinder with radius (perpendicular to z) of 5 km and height (parallel to z) of 10 km, such that the middle of the bottom circle lies at (0,0,-5) and the middle of the top circle at (0,0,5).
- Delete particles if they fly out of the domain.

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## 1 RREA Probability Simulation Set-up

Given an electron with a certain primary energy embedded in a background field, what is the probability that a runaway avalanche occurs?

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### Definition runaway avalanche

- We define a runaway avalanche if there are more or equal than 20 electrons of at least 1 MeV.

### Definition of runaway avalanche probability

- The ratio of single electrons that produce a runaway avalanche over the total number of trials.
- Delete particles below threshold
- Use particle threshold of 1 keV. Positrons may however, depending on their implementation, still go into annihilation.

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### Domain

- The domain should be sufficiently large, such that no particle reaches the boundary

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### Initial energy

- The initial energy ranges between 10 keV and 1 MeV. Use a log spaced grid of at least 50 points.

### Electric fields

- Use electric fields ranges between  $2 \cdot 10^5 \text{ V/m}$  and  $30 \cdot 10^5 \text{ V/m}$ . Use a linear spaced grid of least 50 points.
- We will produce two results. One with the field such that the initial electron is accelerated, one with the field such that the initial electron decelerates. (So +/- parallel to initial momentum)

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## 2 RREA Characteristics Simulation Set-up

### Initial seed

- Every simulation starts with 200 electrons of 100 keV.
- Particle start always in the center, i.e.  $(x,y,z) = (0,0,0)$ , which their momentum directed upwards (positive  $z$ ),  
5 parallel with the electric field such that they are accelerated by the field.

### Energy threshold

- Take an energy threshold of 10 keV, meaning that all particles below 10 keV do not appear on the output.
- If your code is using friction for part of the spectrum, use 10 keV as the point to separate the part of friction and the part of explicit collisions.

### 10 Electric fields

- In this work we consider 12 electric fields:  $[6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28] \times 10^5 V/m$ .
- The field is oriented in the  $-z$  direction, such that the electrons are accelerated by the field upwards.

### Time limit

- To avoid very large file size, a time limit must be set depending on the electric field.
- 15 –  $t_{\text{stop}} = [1078, 522, 351, 277, 233, 199, 179, 164, 153, 144, 135, 130]$  nanoseconds; each value corresponding to an electric field (following the same order).

### Simulation type 1 - evolution in time

Output all particles at 32 times :  $t = [14., 26., 39., 51., 64., 78., 90., 102., 116., 124., 135., 144., 153., 164., 179., 199., 215., 233., 262., 277., 290., 312., 351., 406., 464., 479., 522., 599., 719., 838., 958., 1078.]$  nanoseconds; regardless of  
20 position.

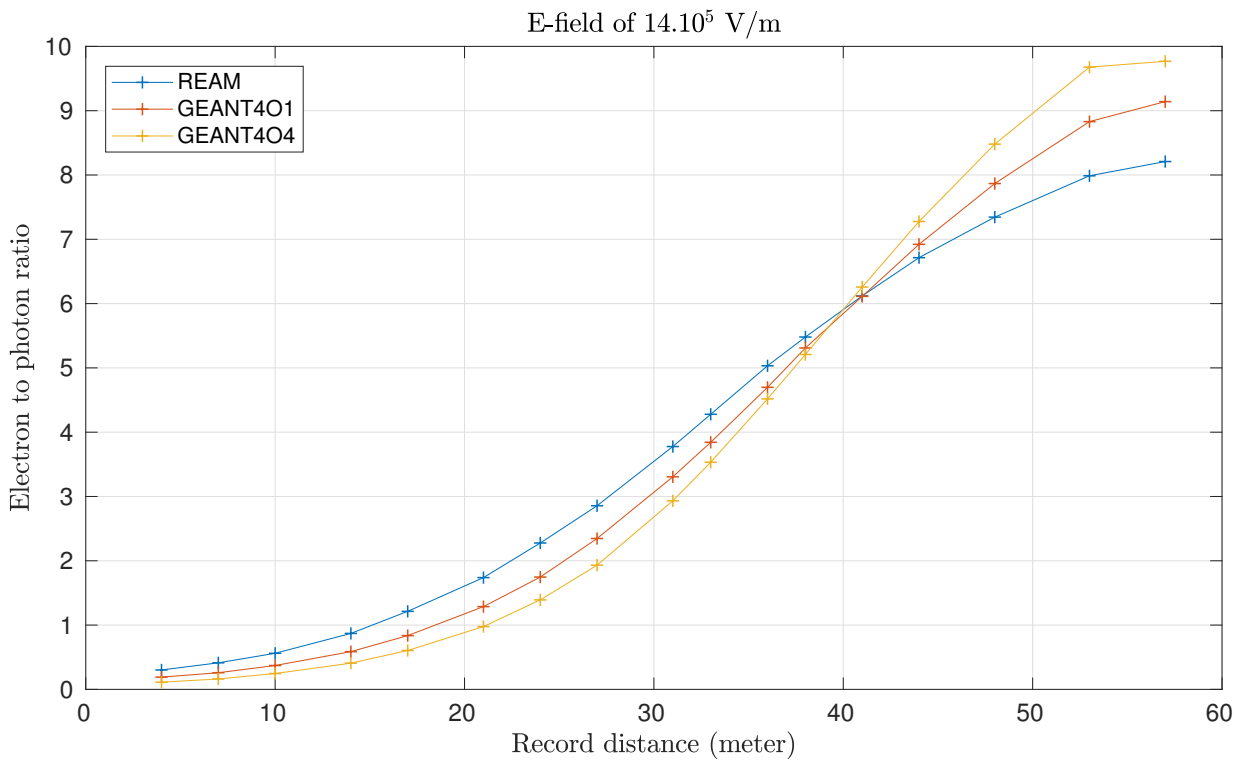
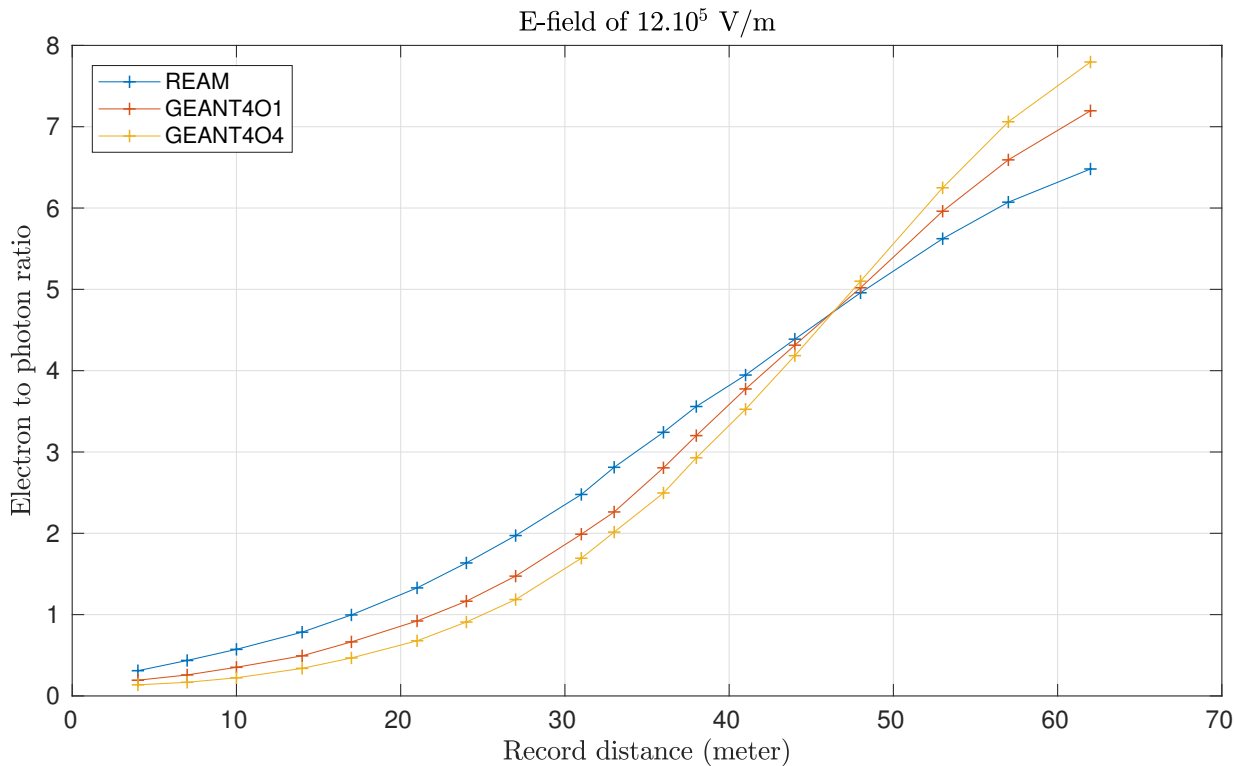
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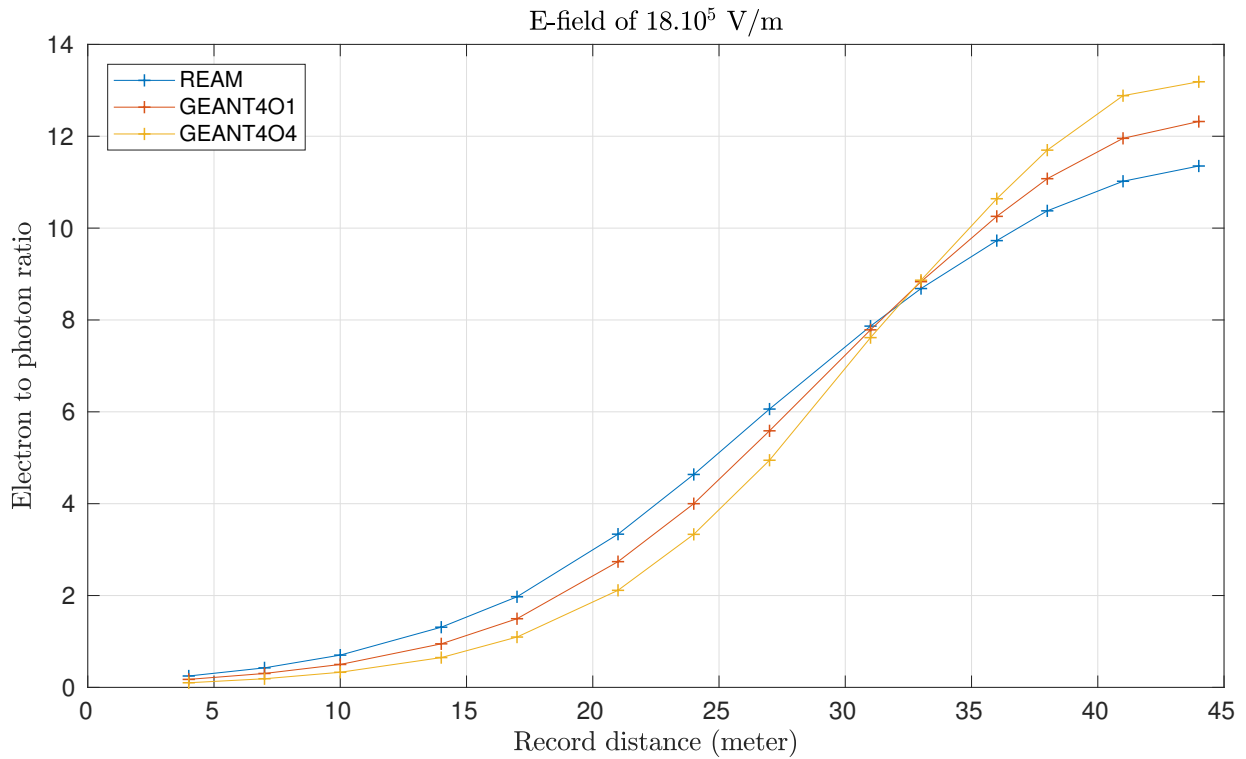
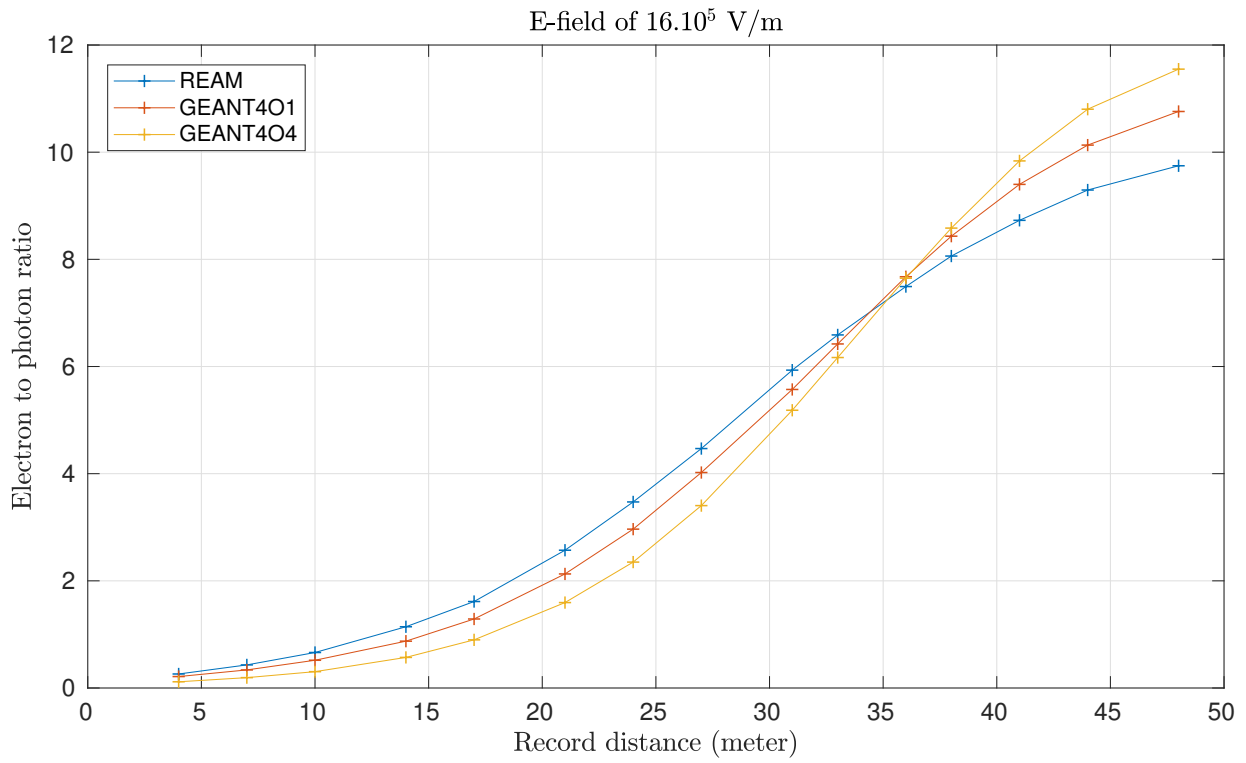
Output all particles at 32 distances :  $z = [4., 7., 10., 14., 17., 21., 24., 27., 31., 33., 36., 38., 41., 44., 48., 53., 57., 62., 70., 74., 77., 83., 94., 108., 124., 128., 139., 160., 192., 224., 256., 288.]$  meters, regardless of time of arrival.

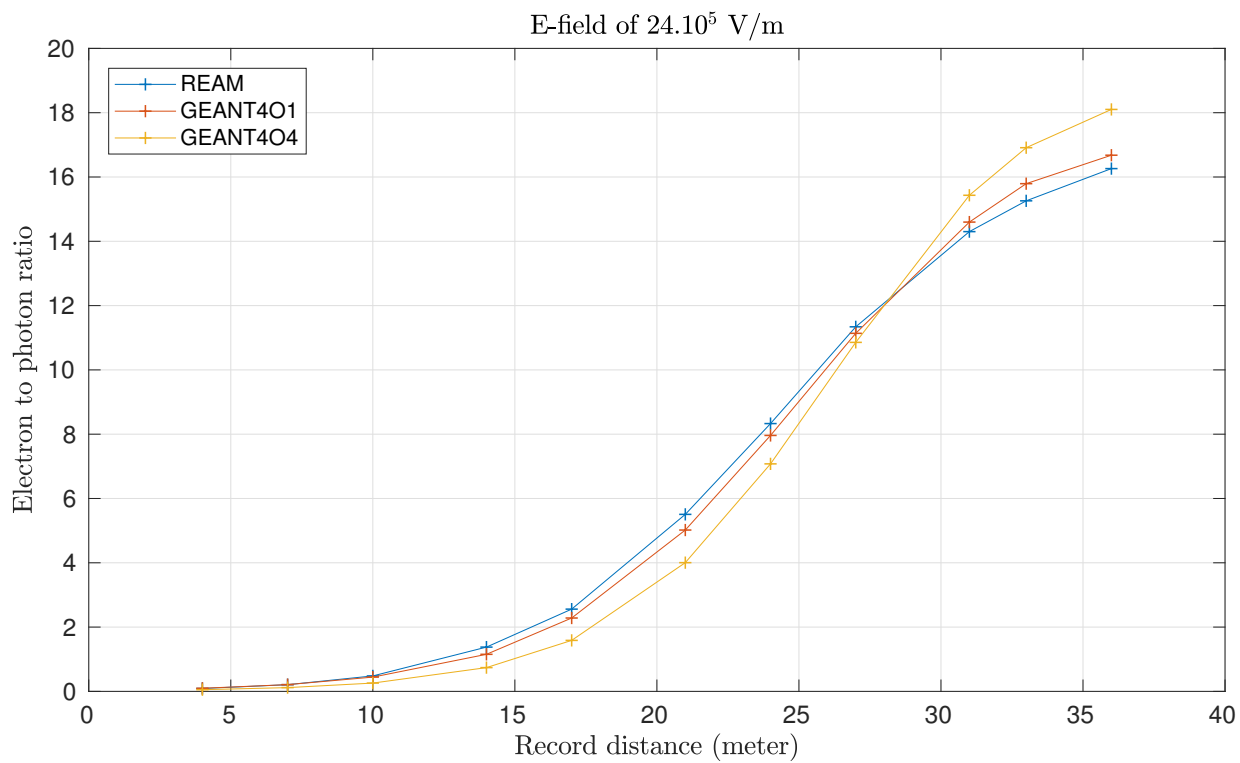
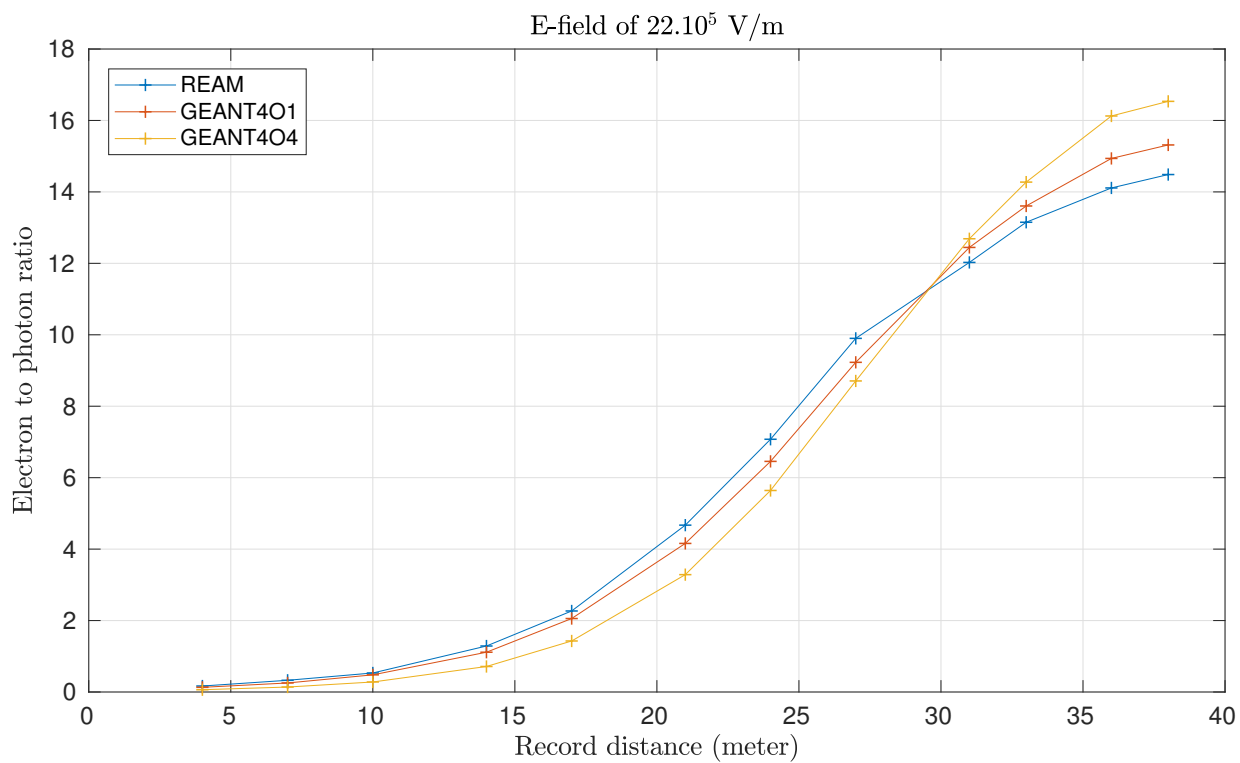


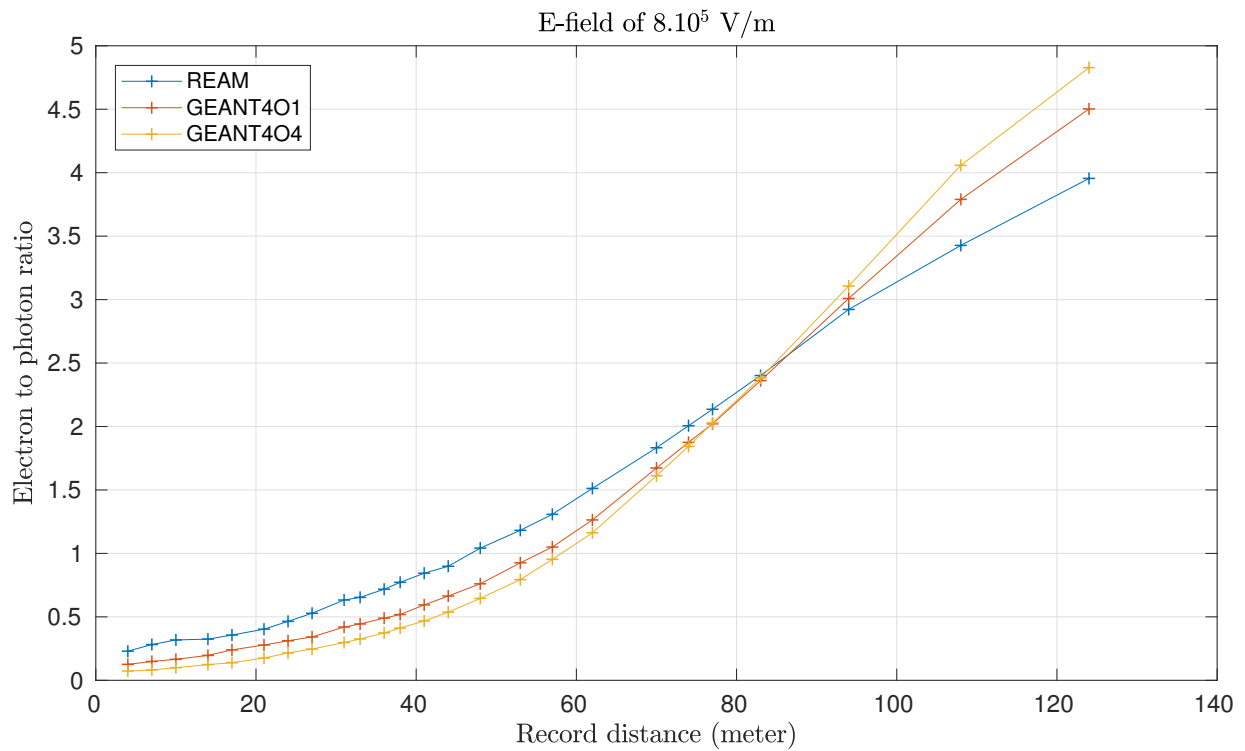
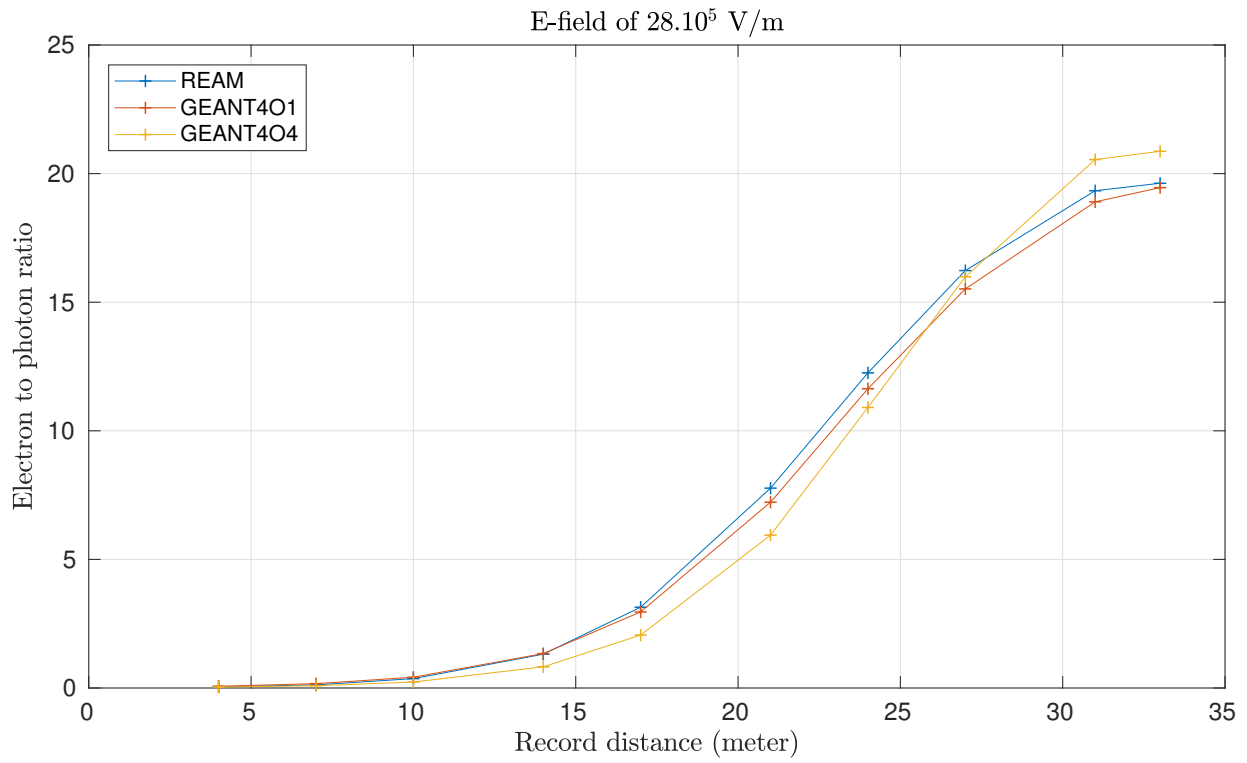
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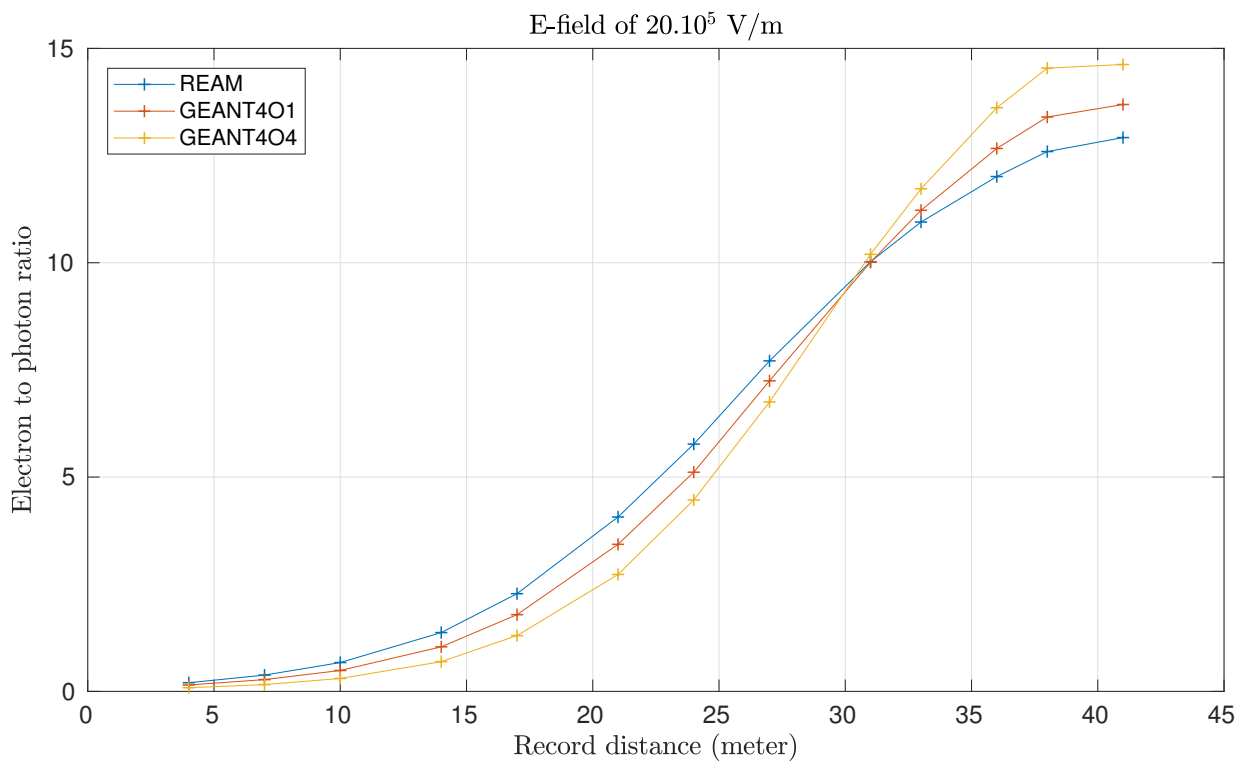
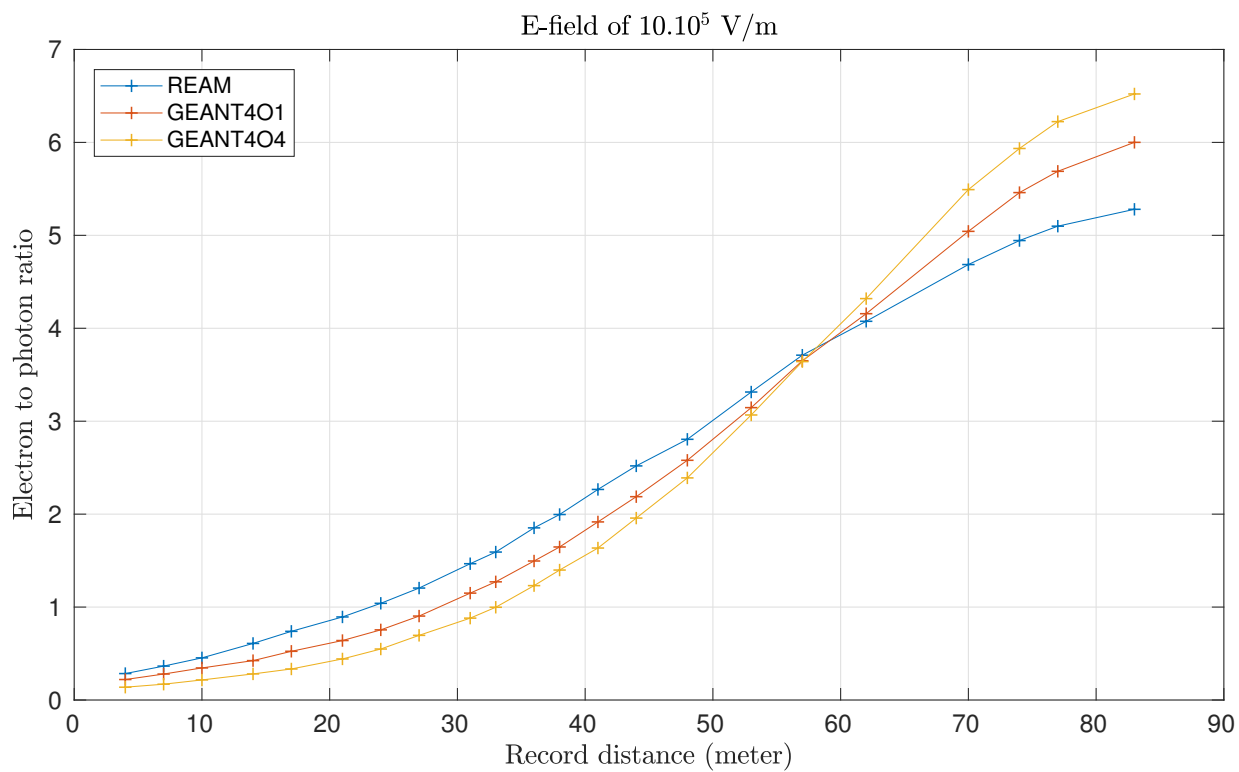
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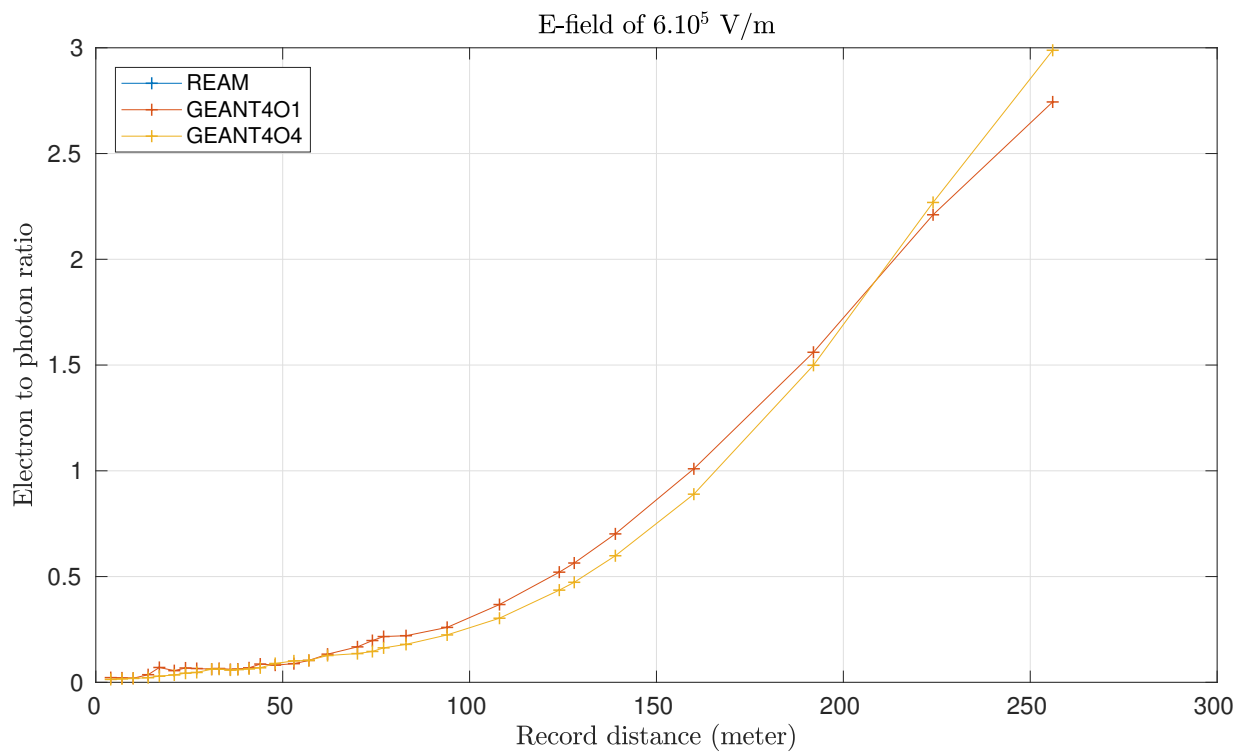
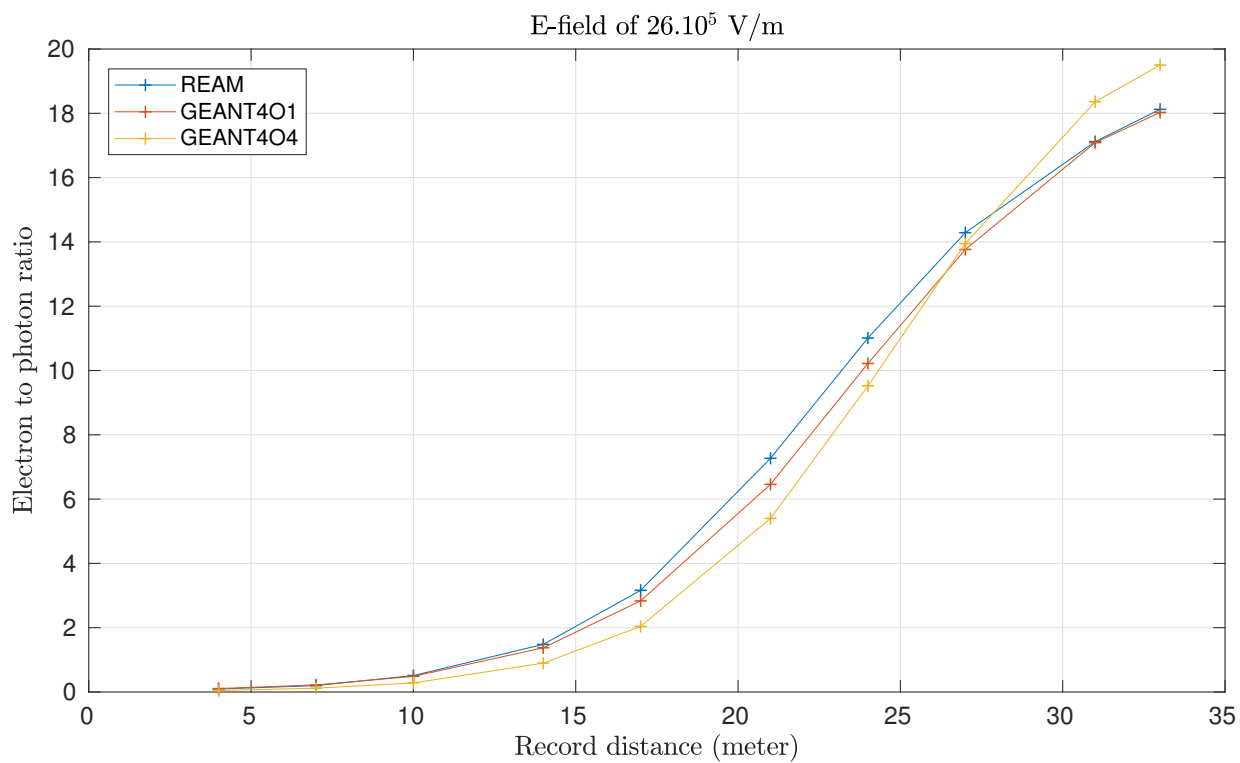




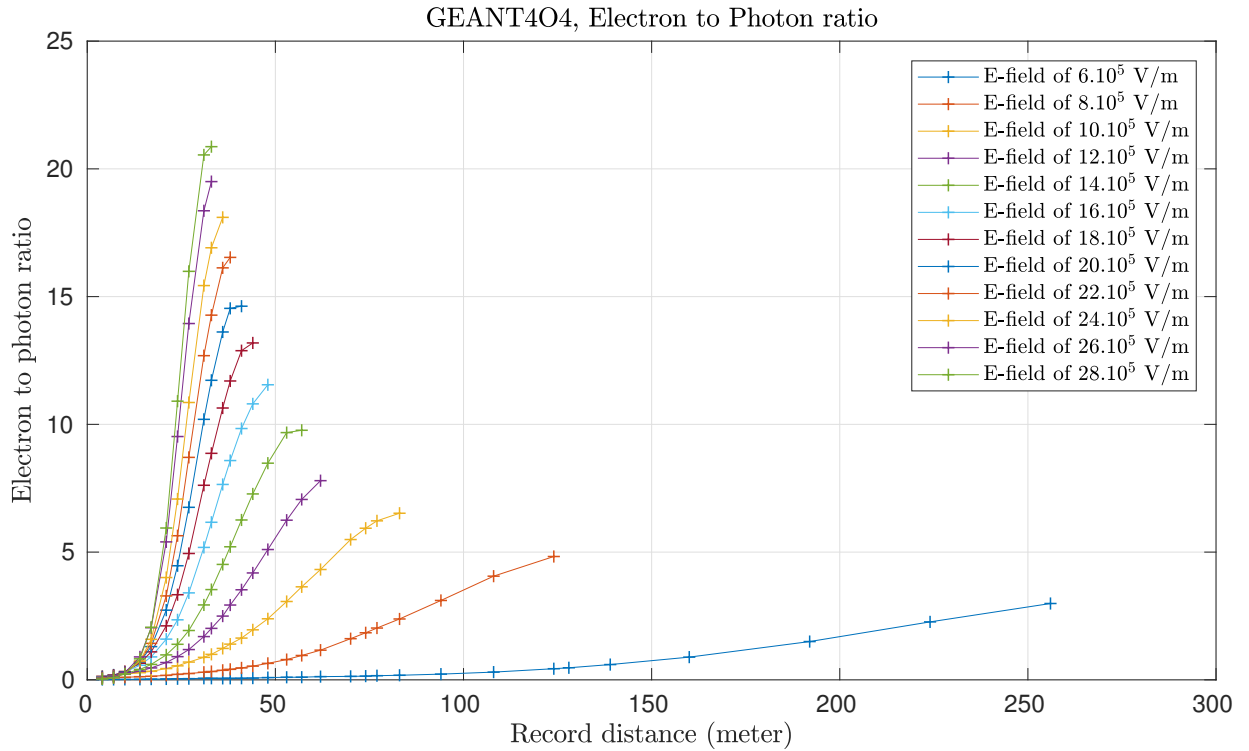
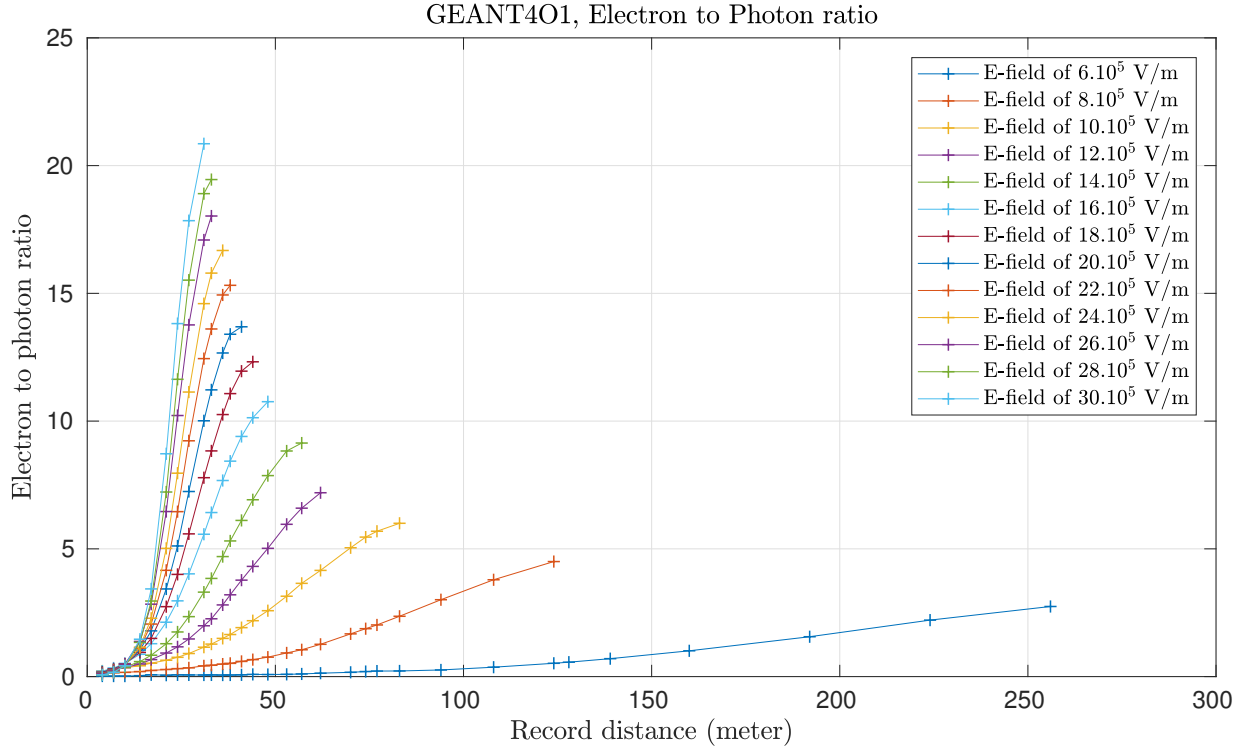


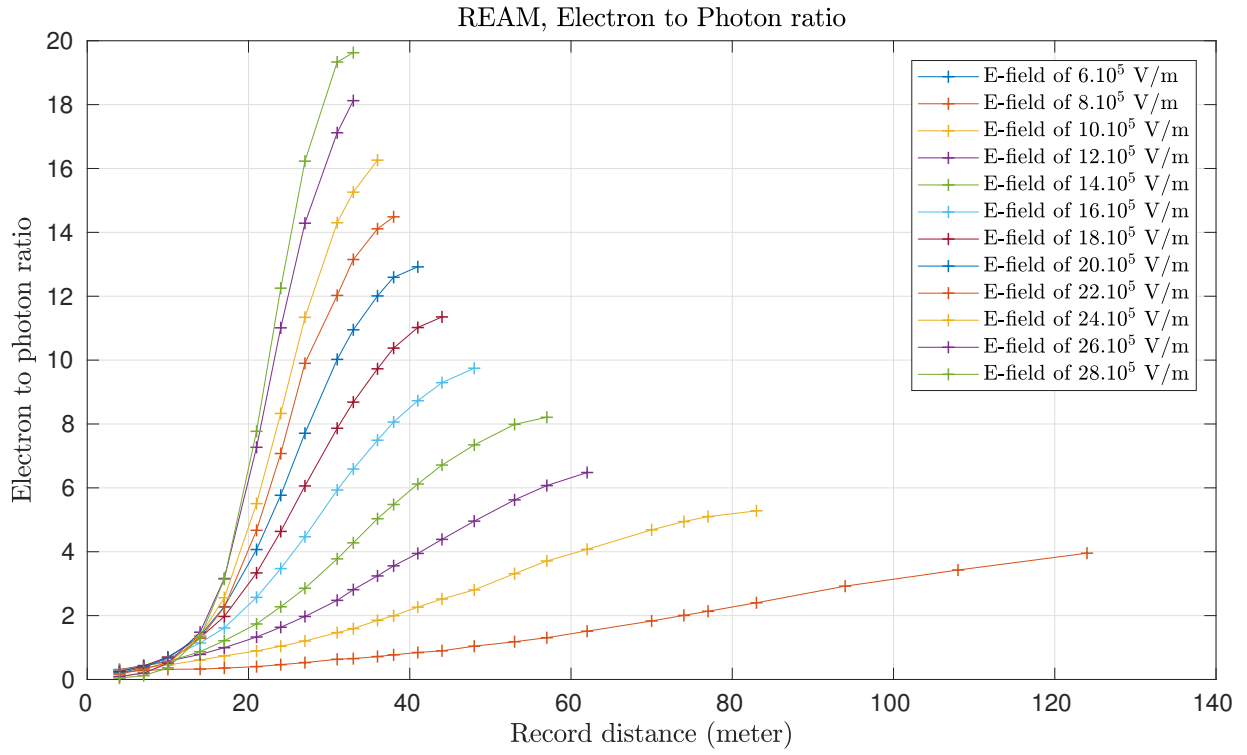






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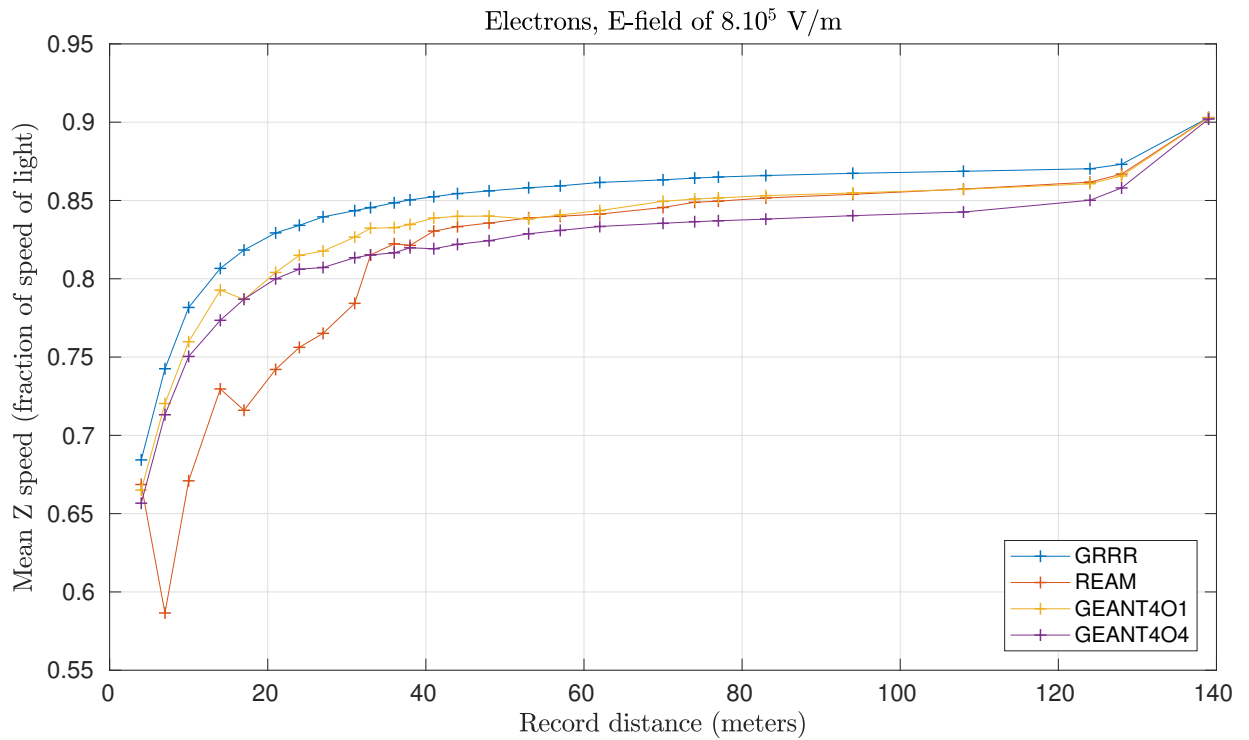
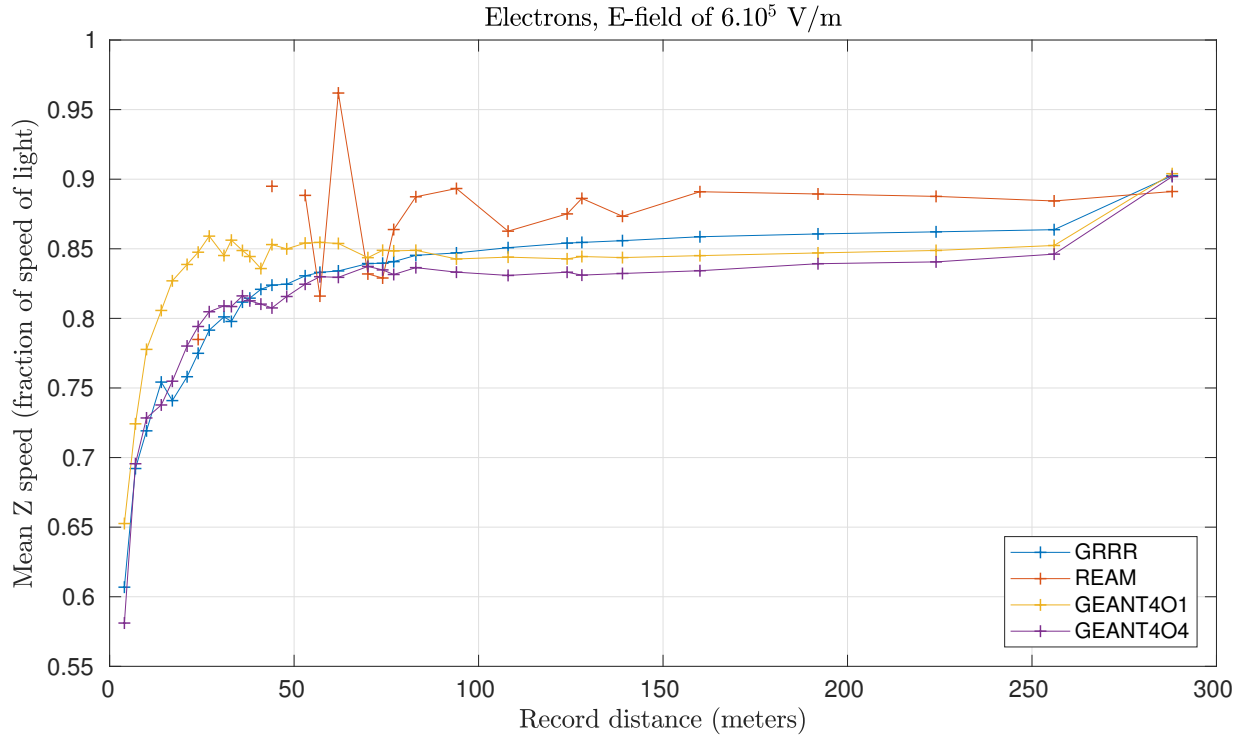


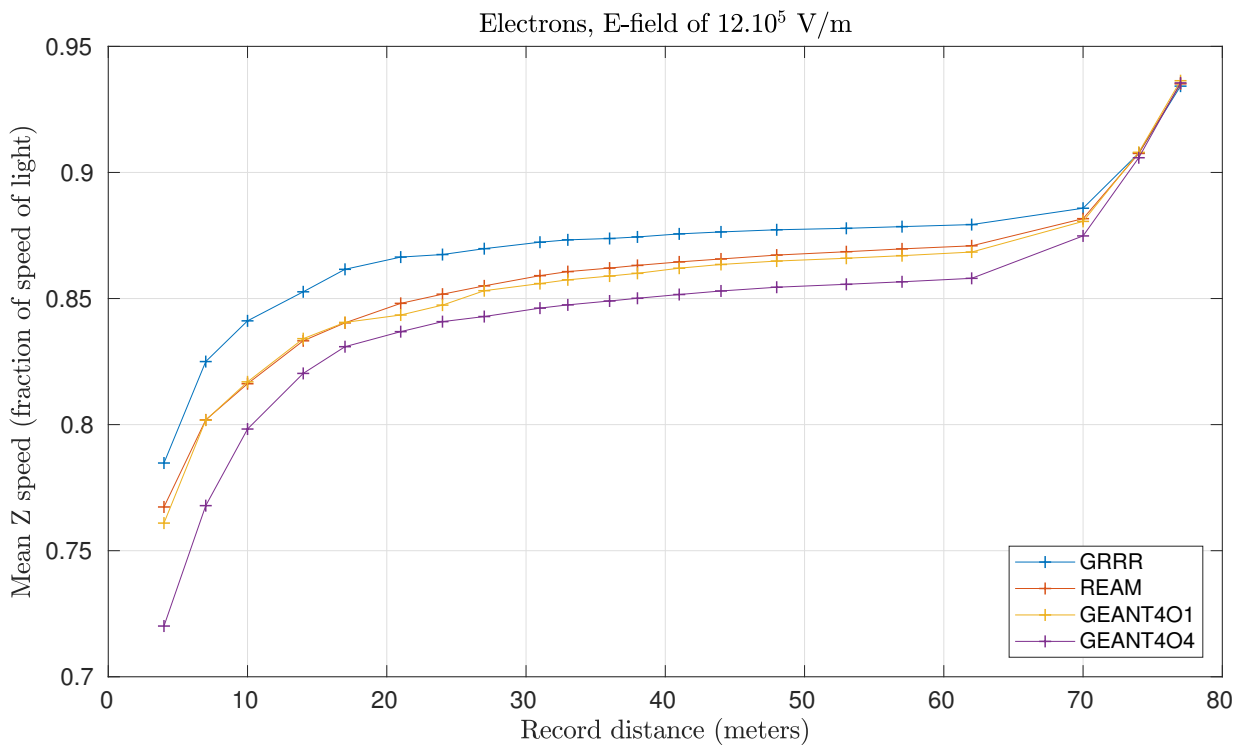
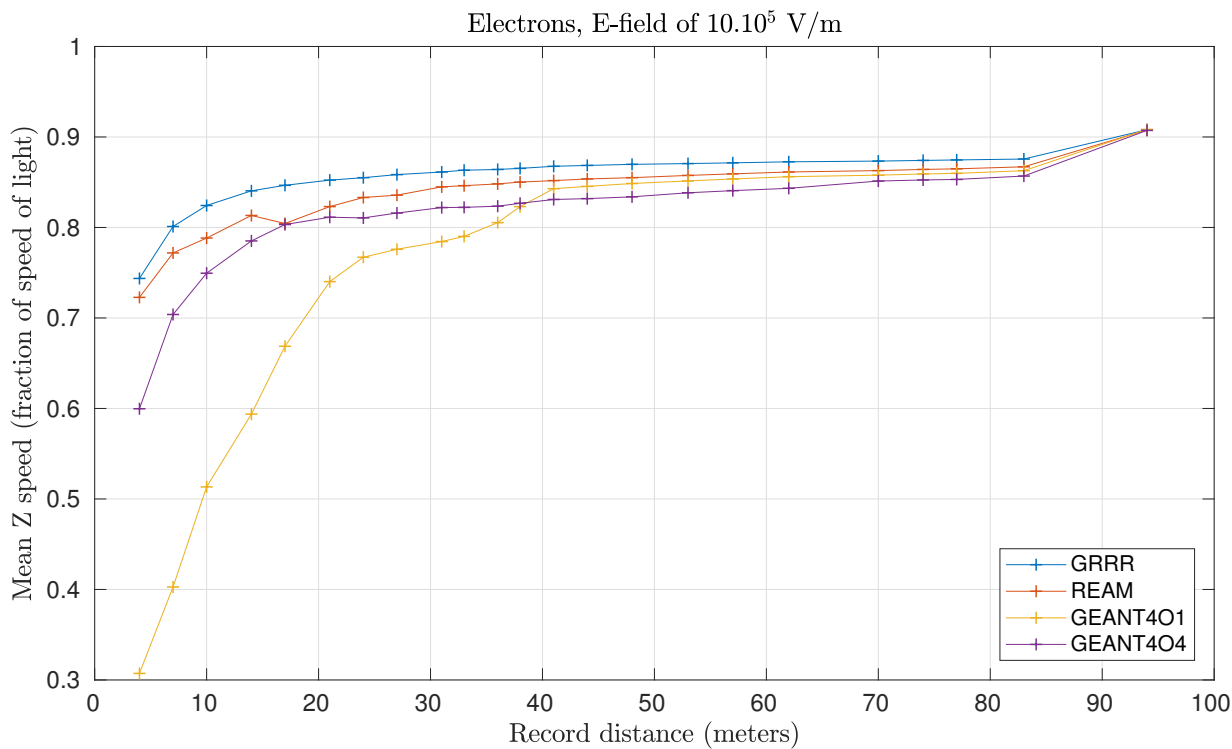


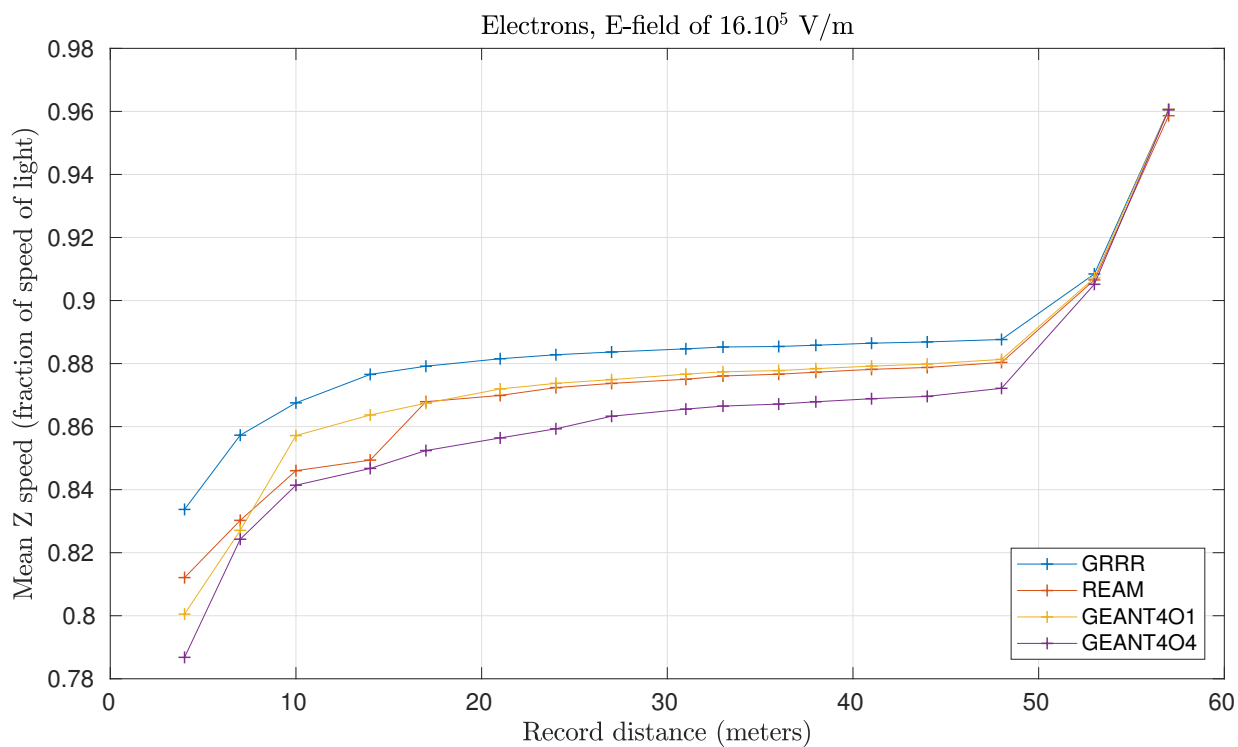
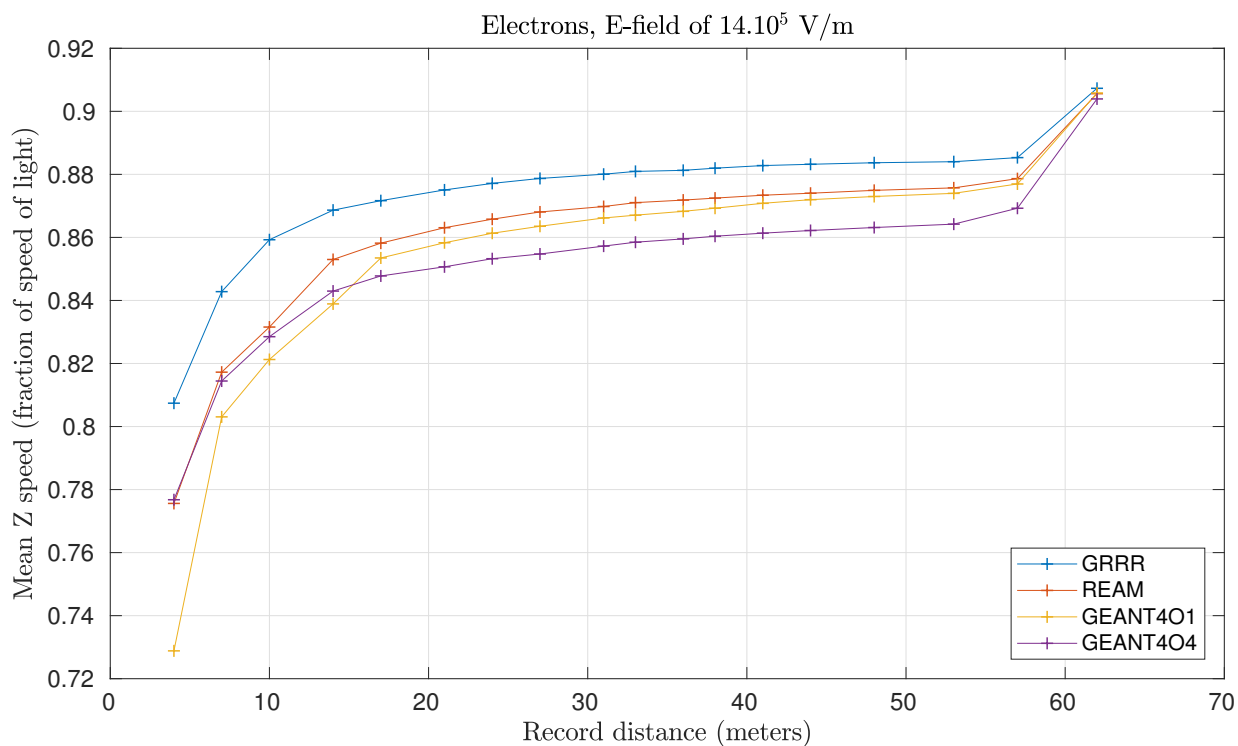


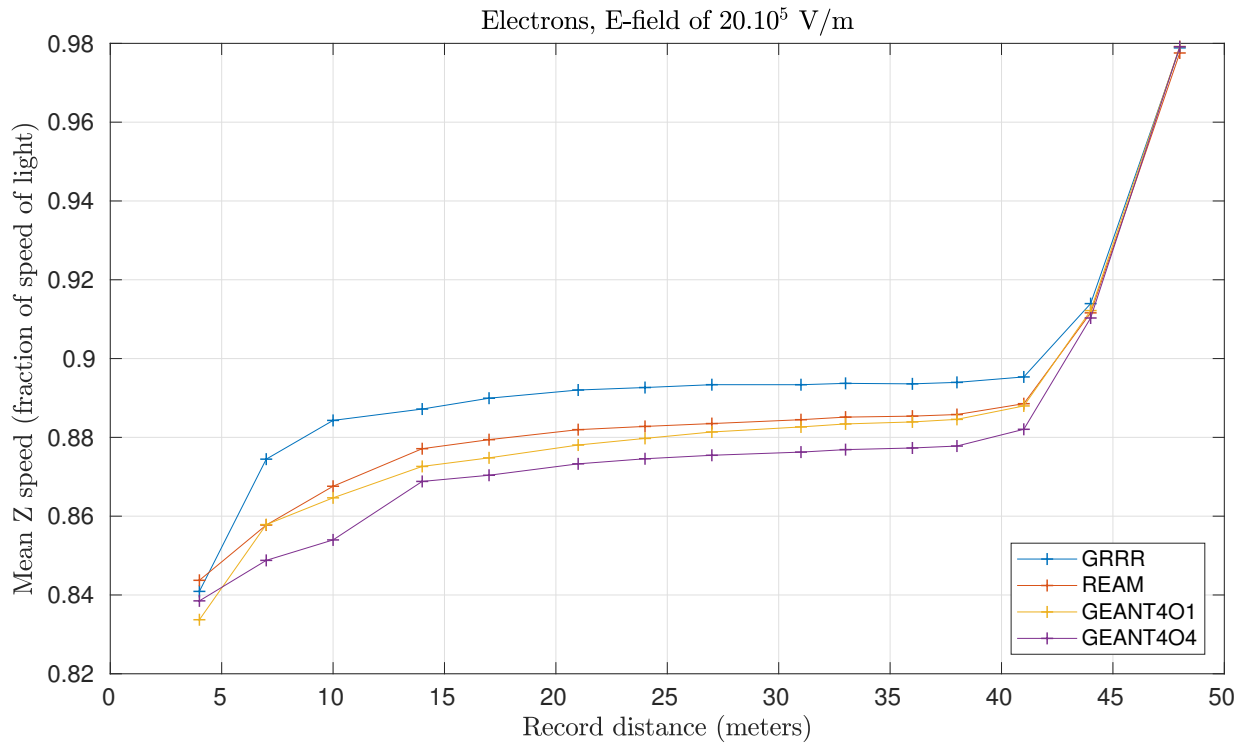
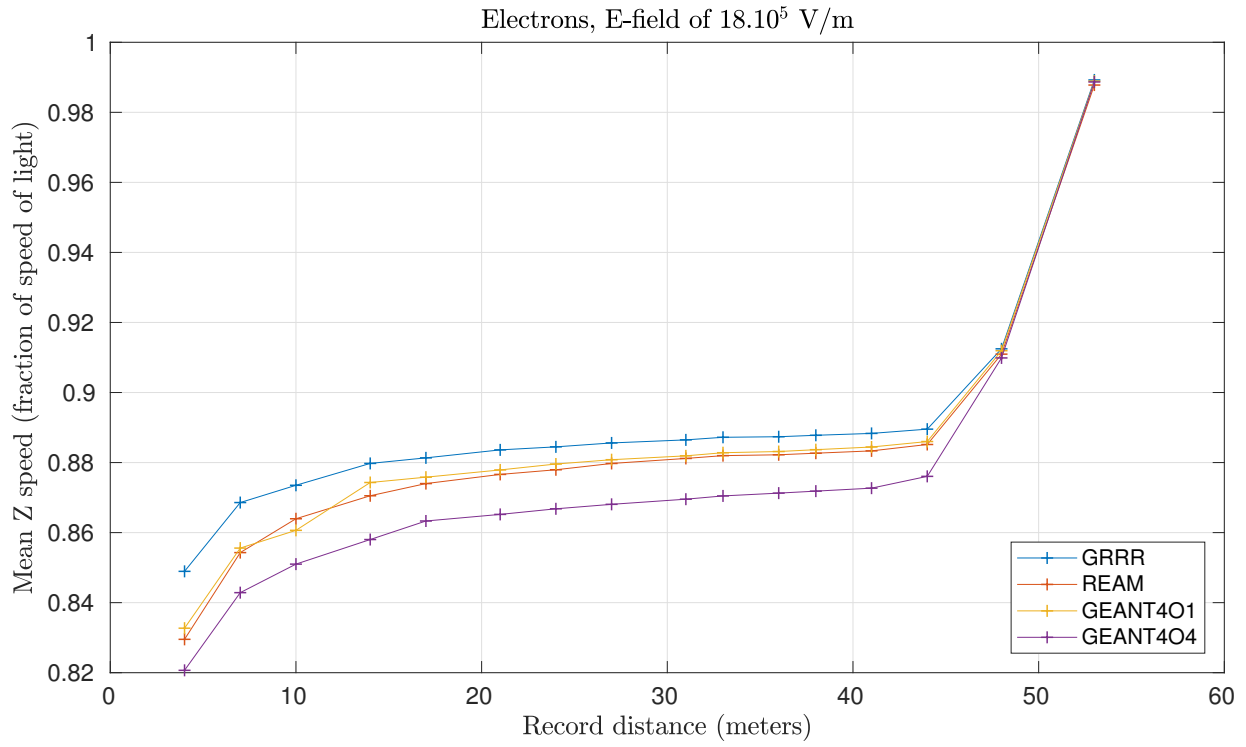
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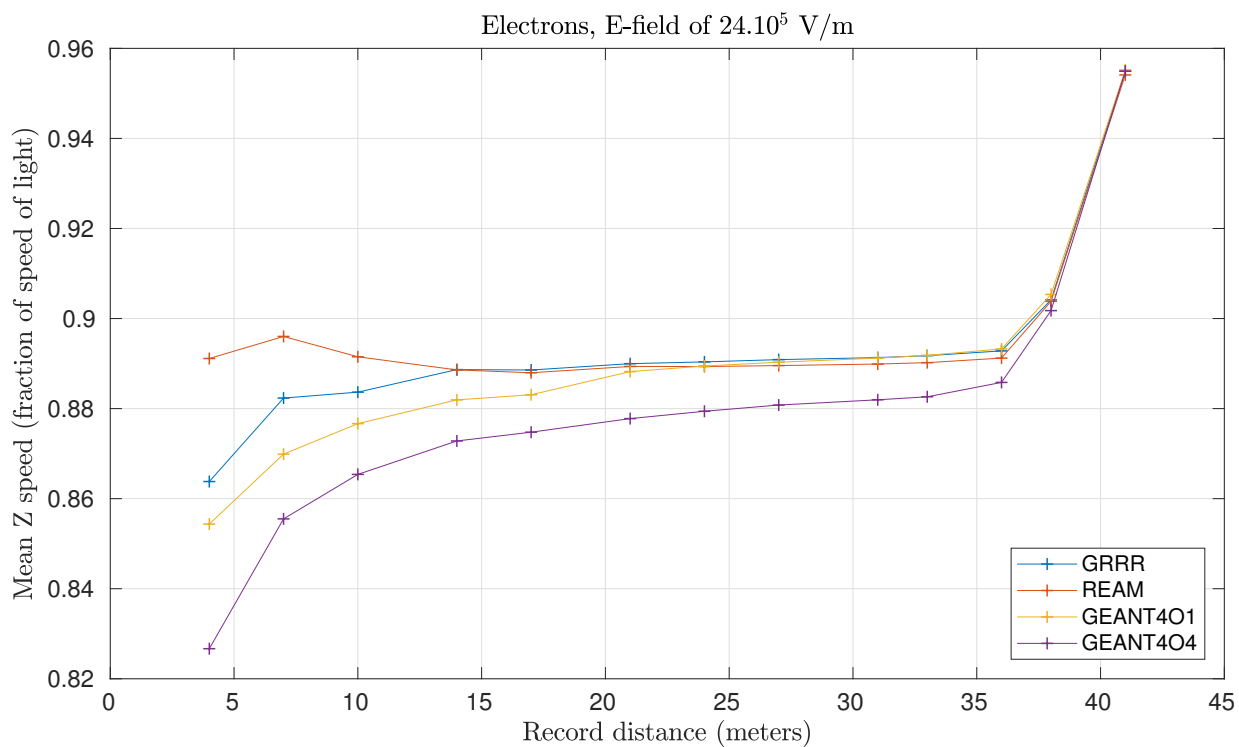
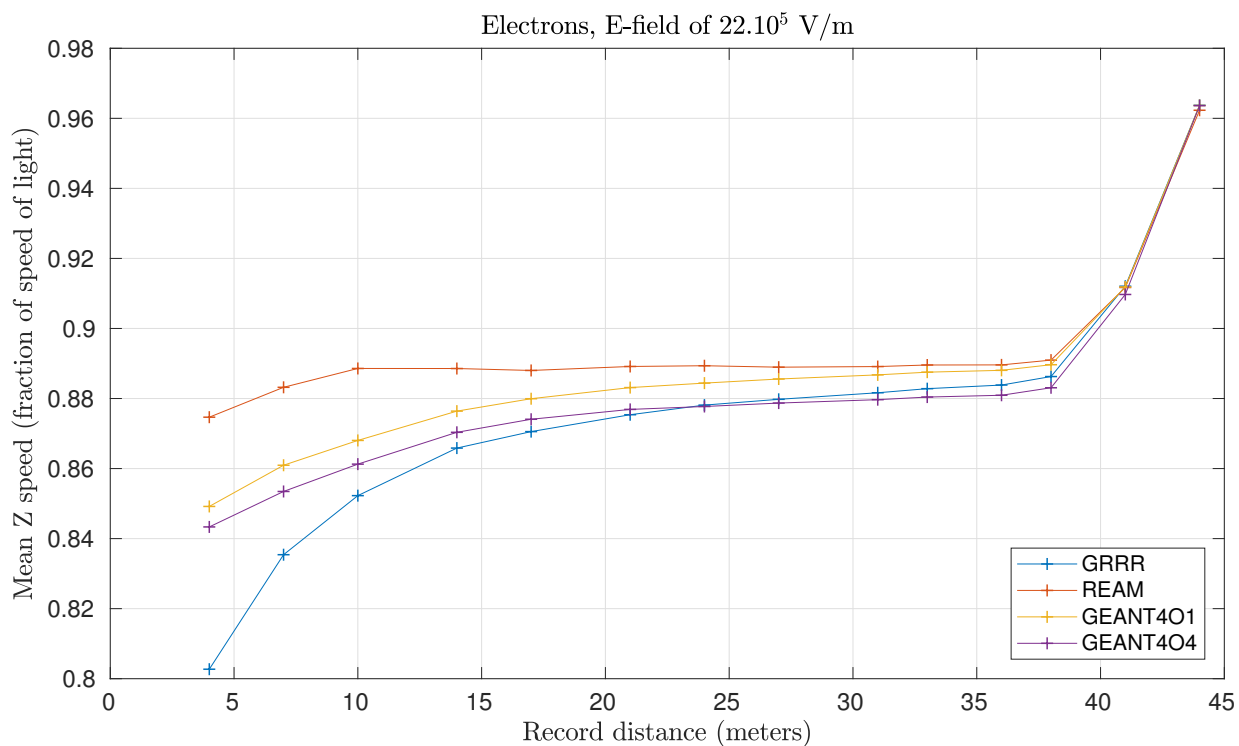
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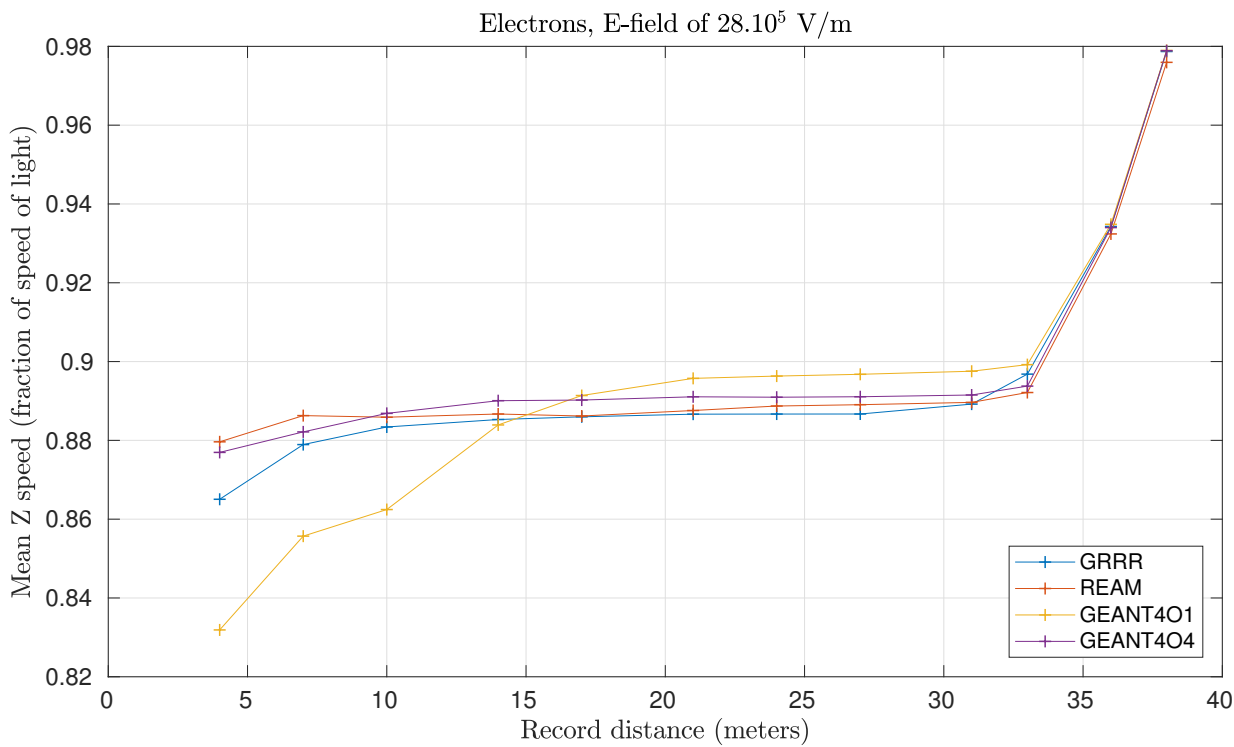
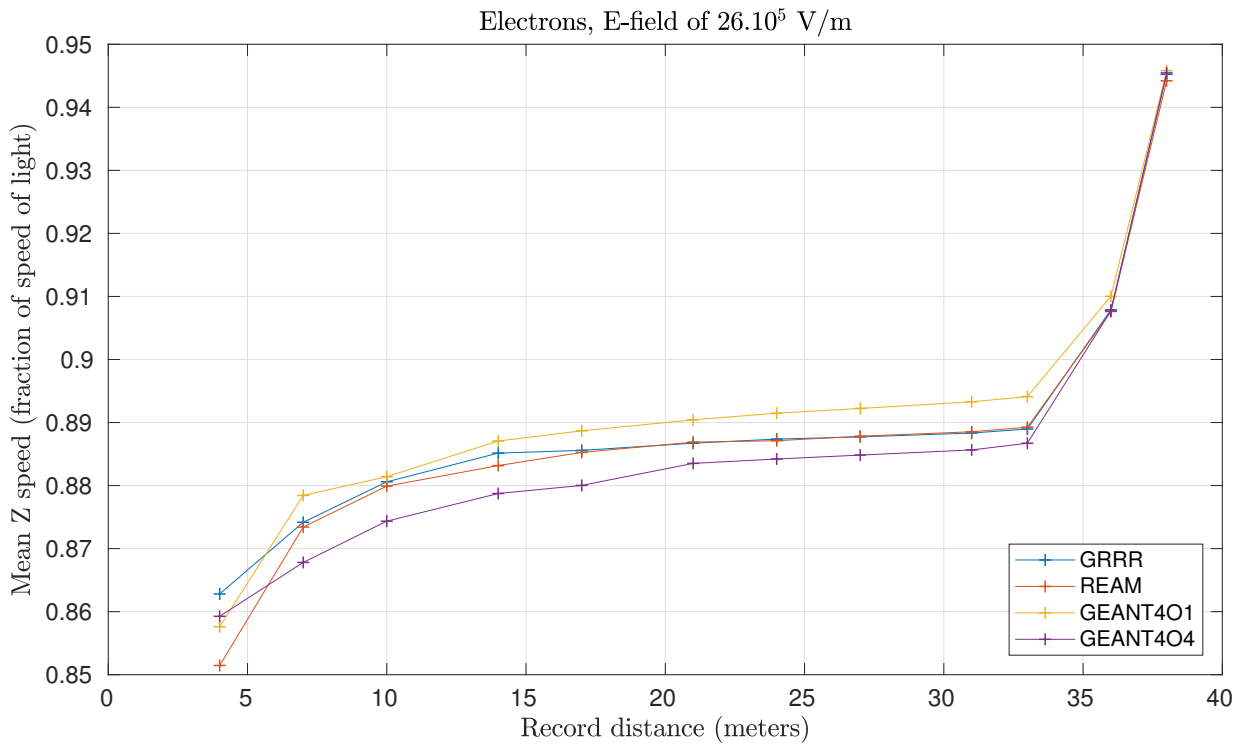




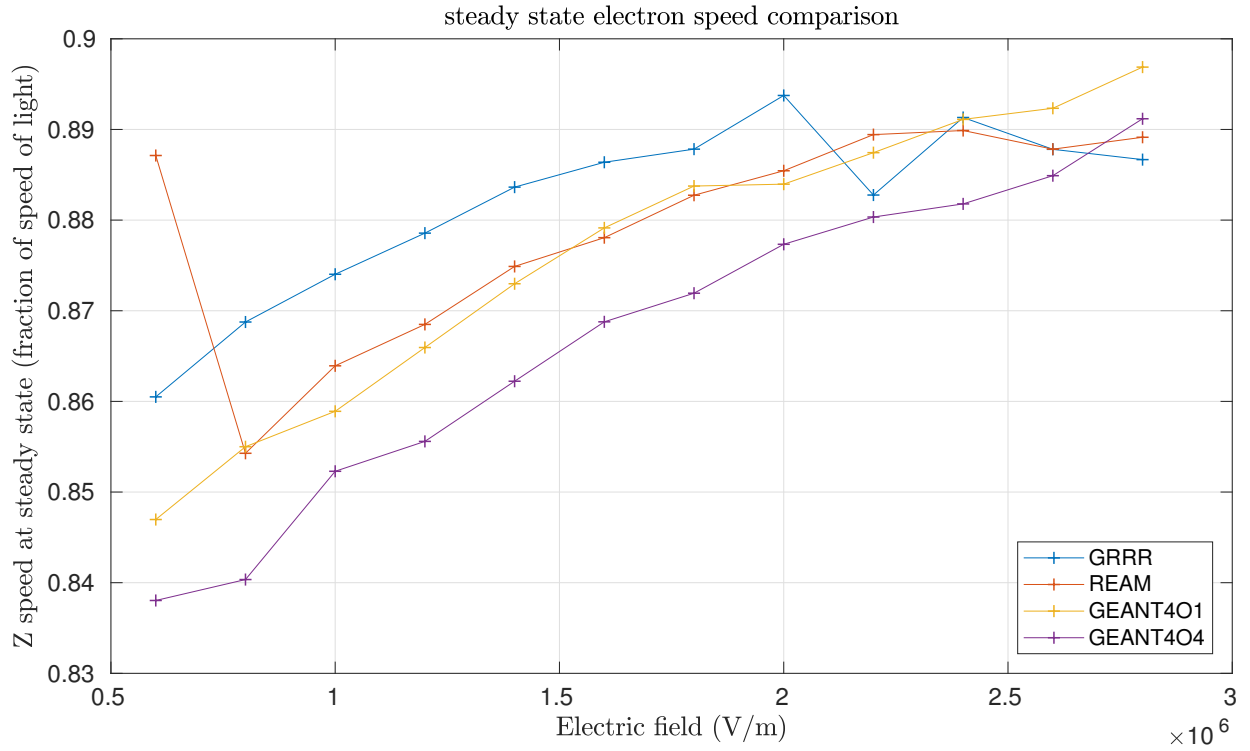




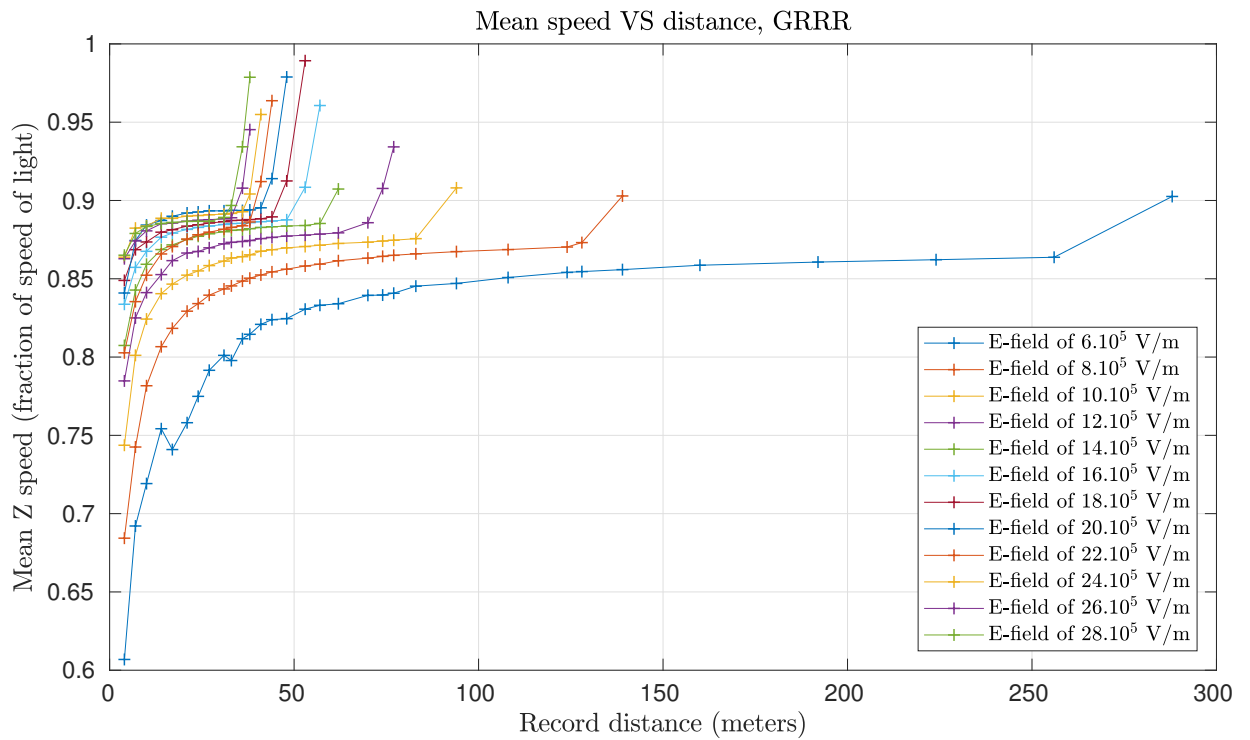


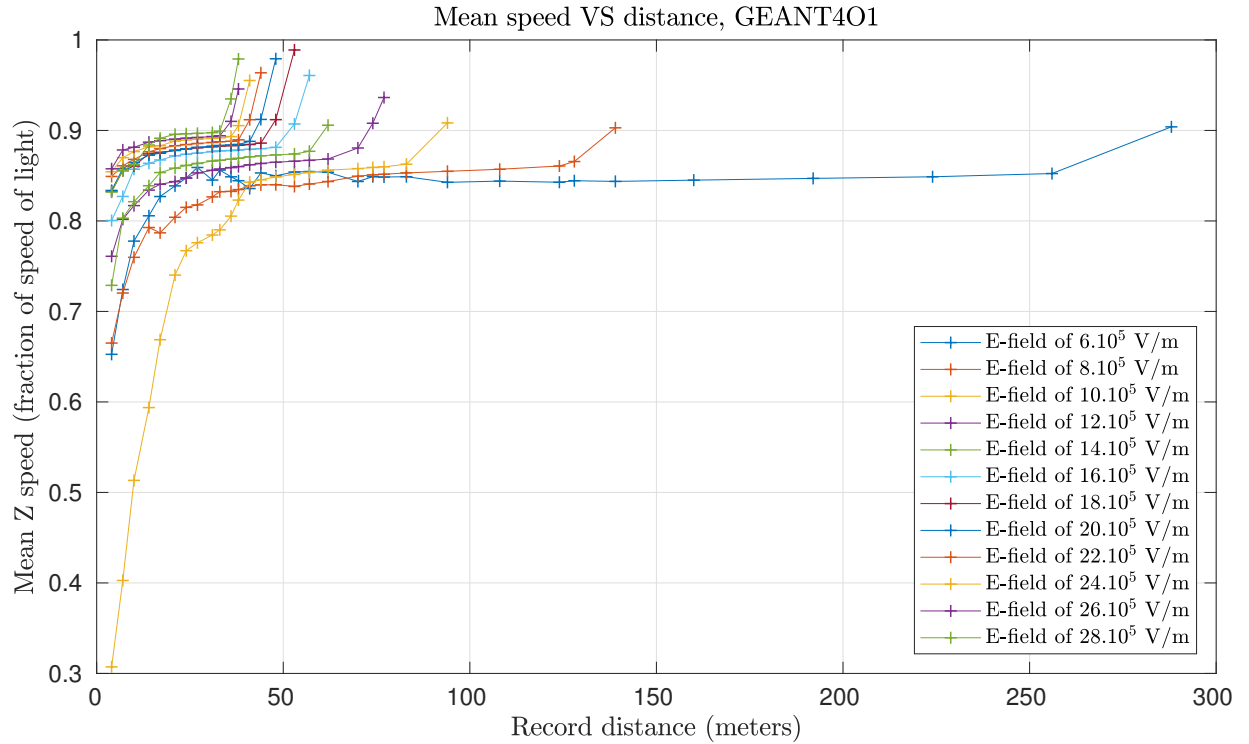
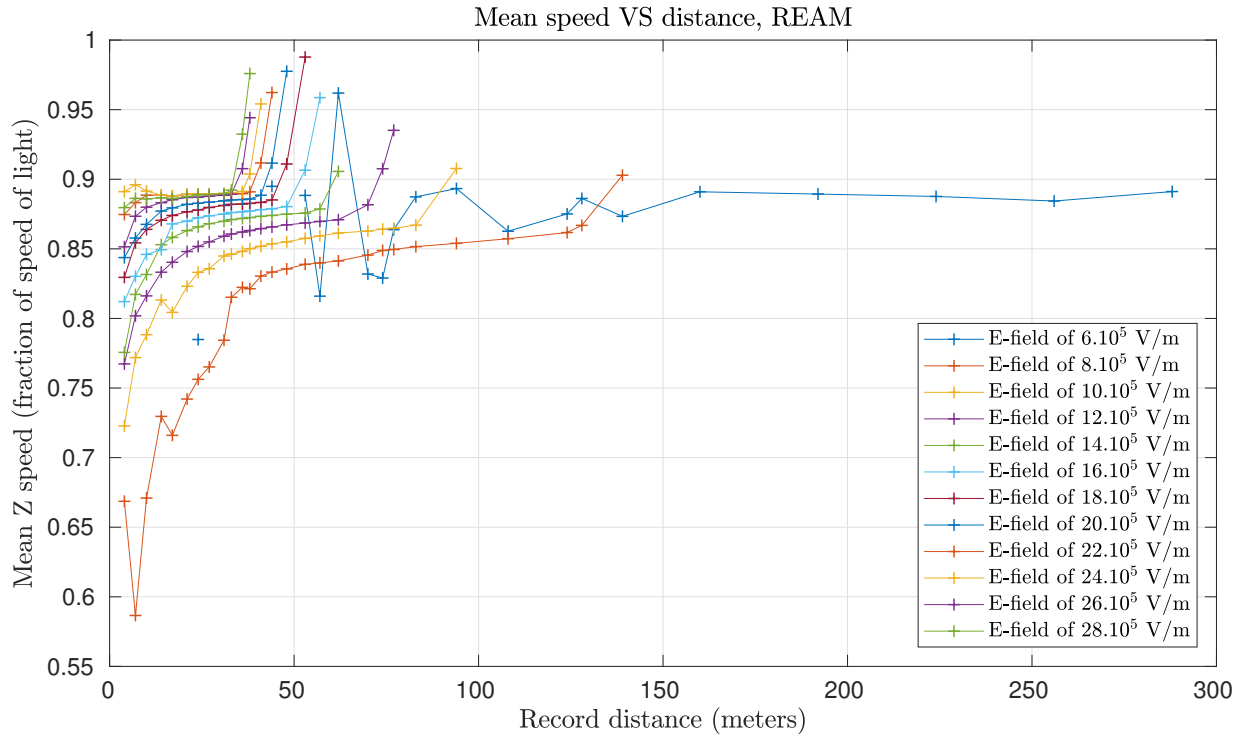


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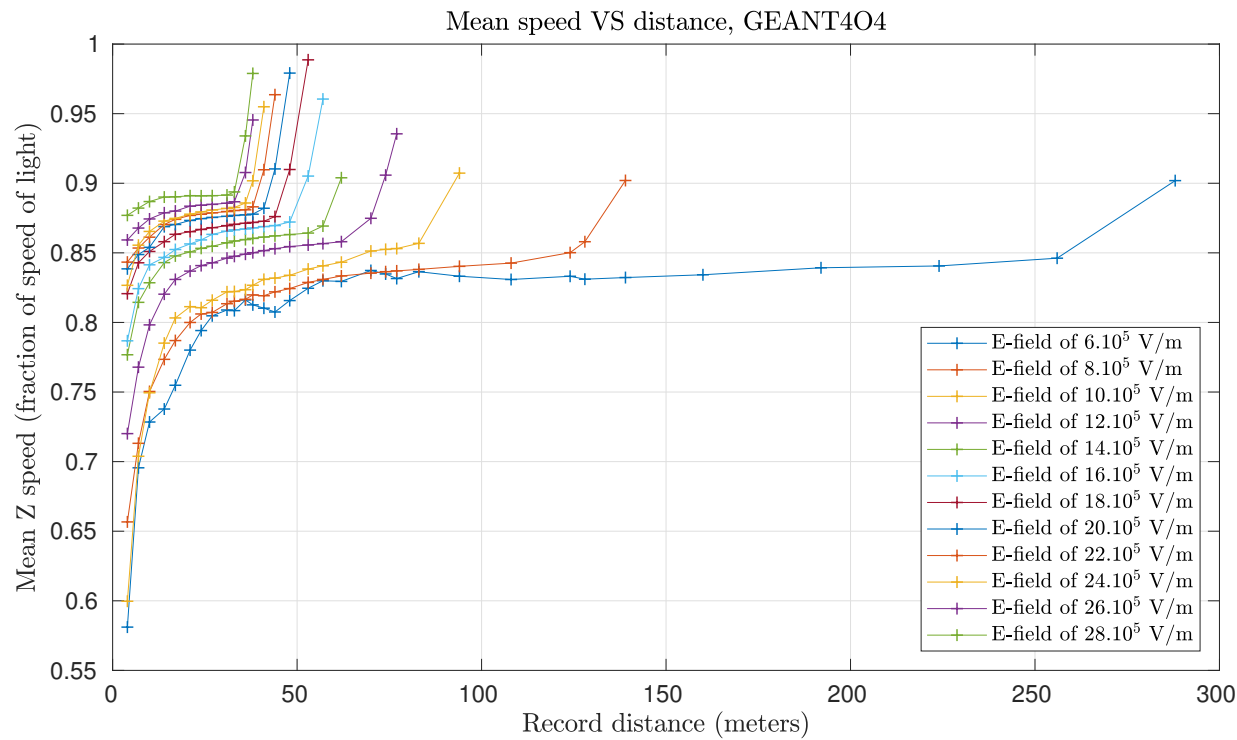


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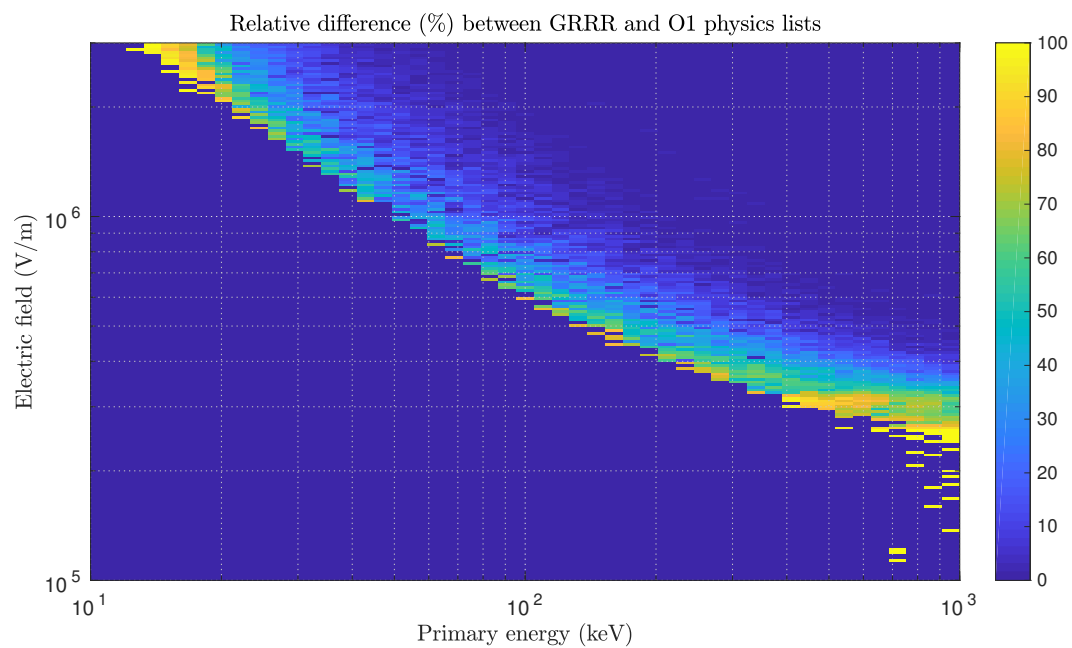
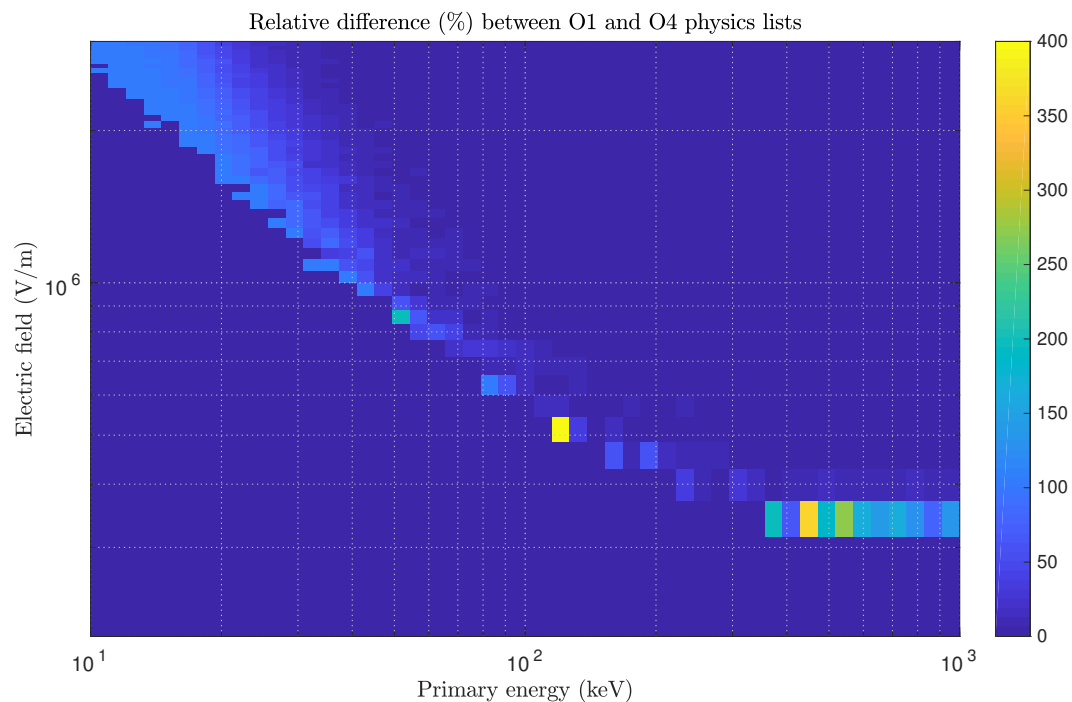


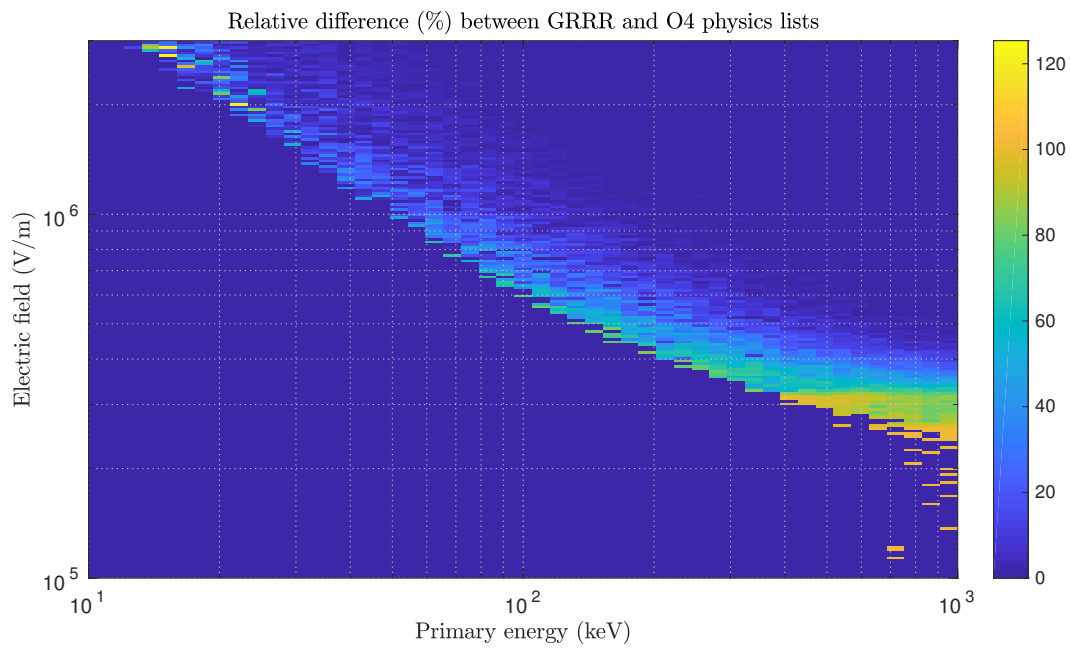




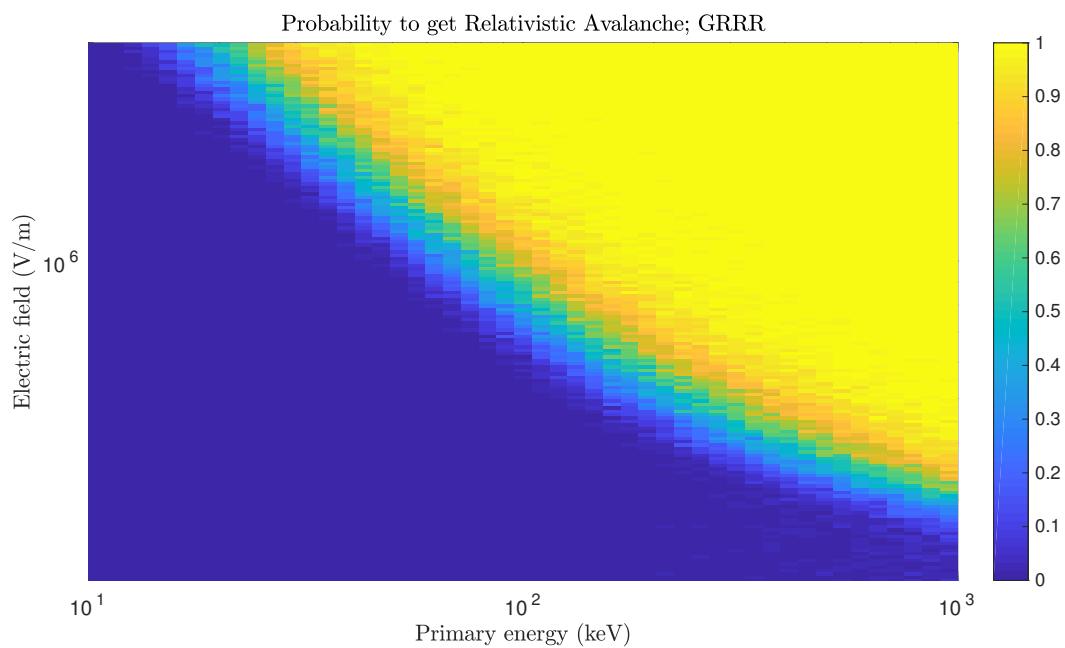
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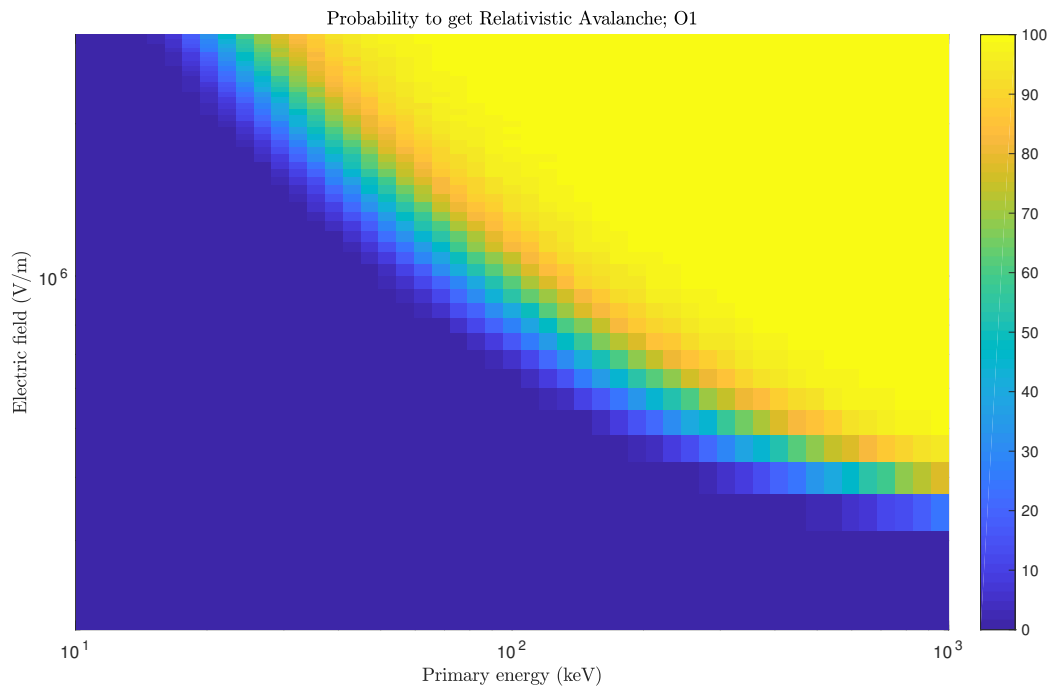
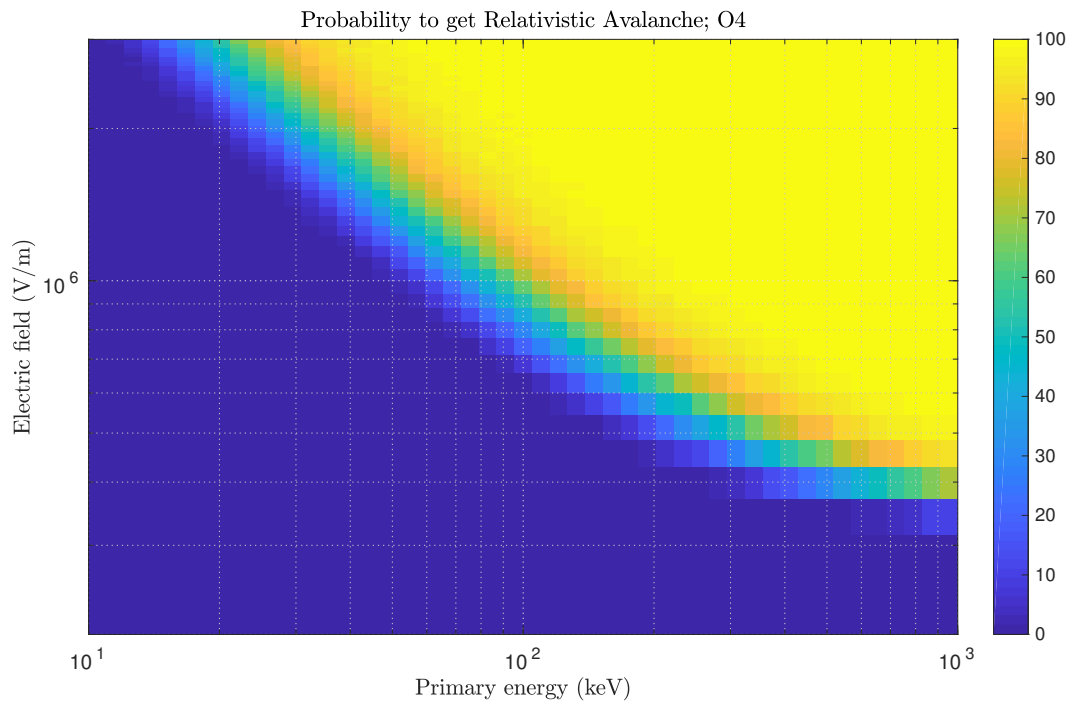
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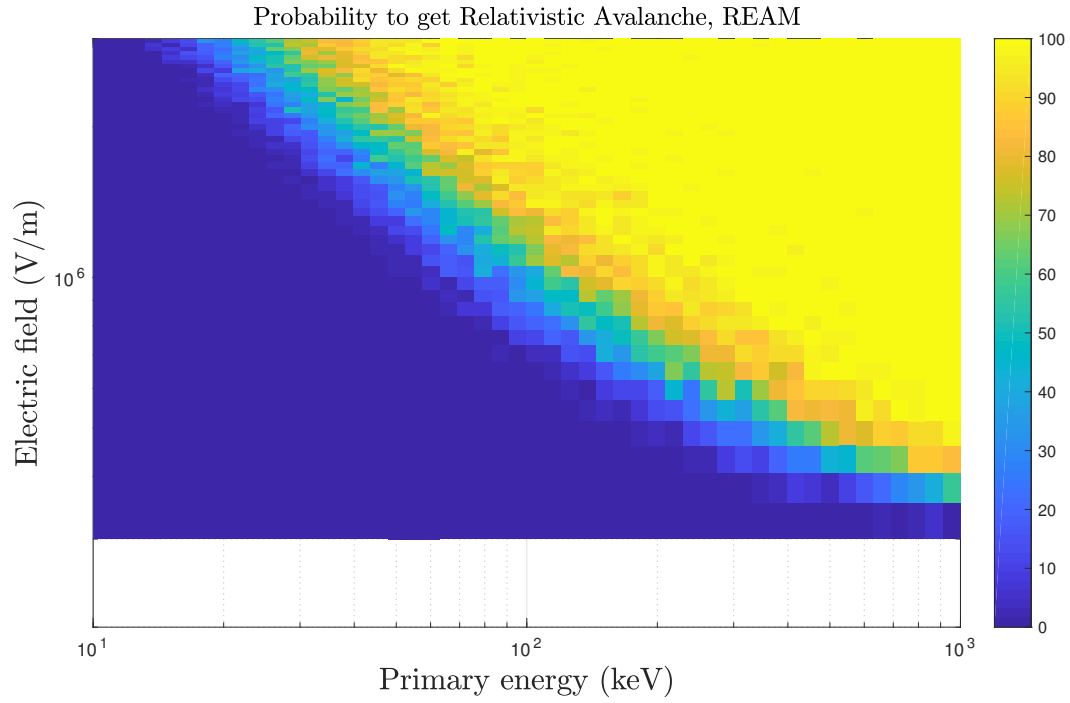




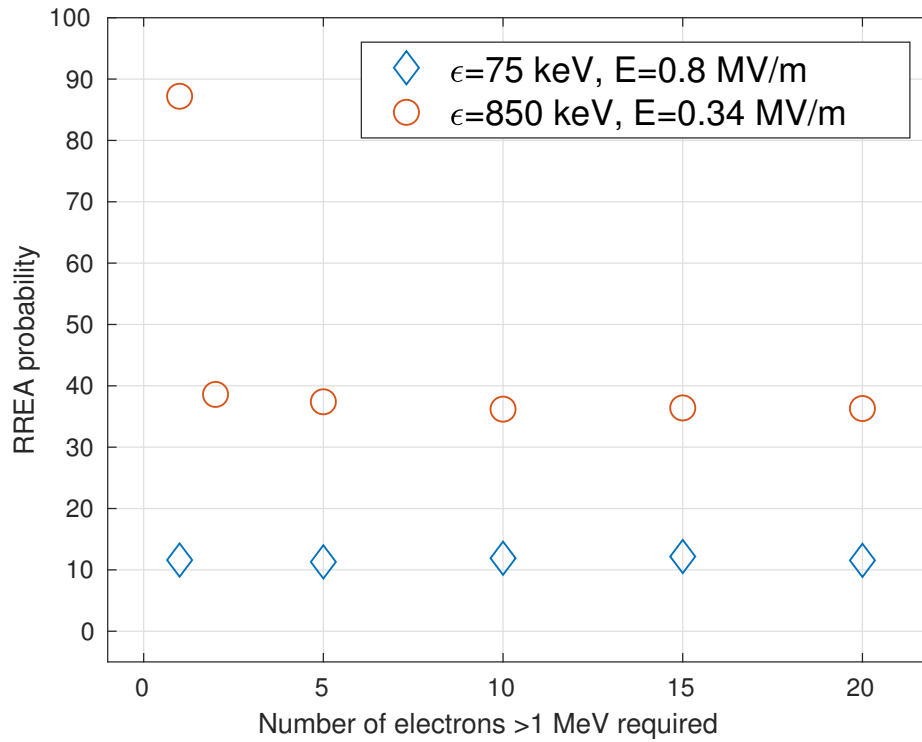
## 5.2 Code By Code





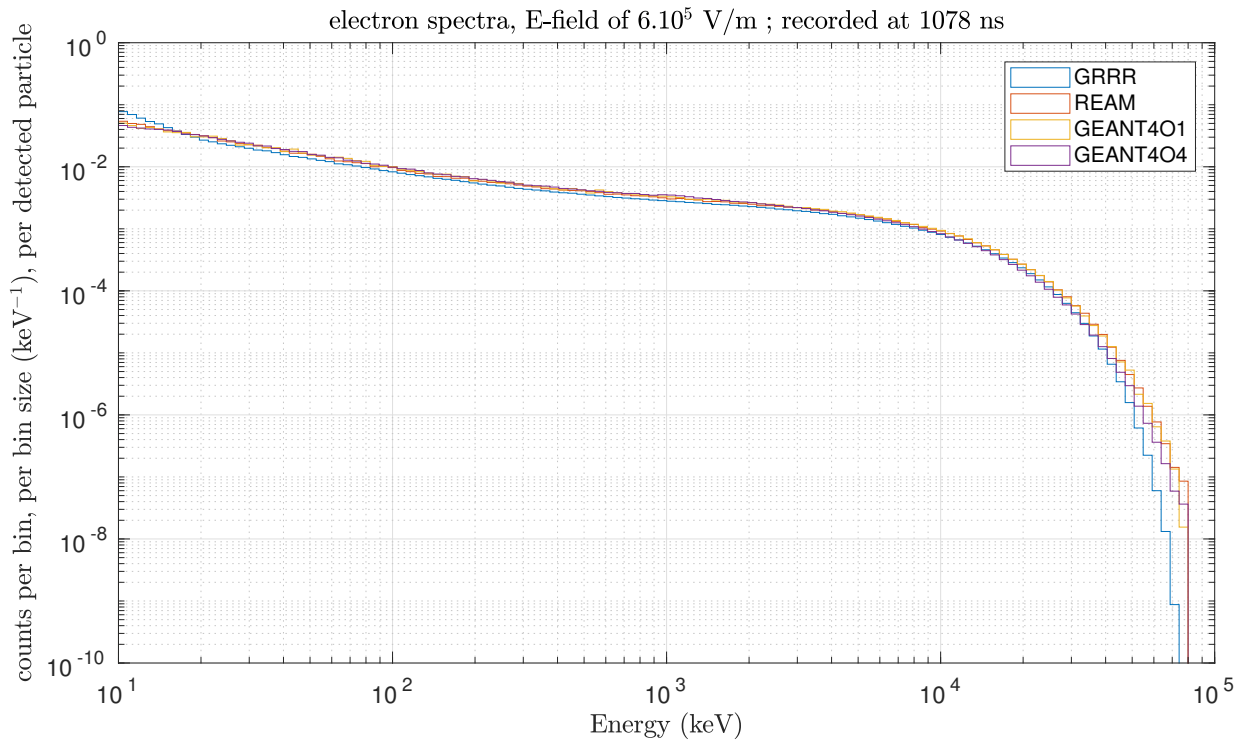
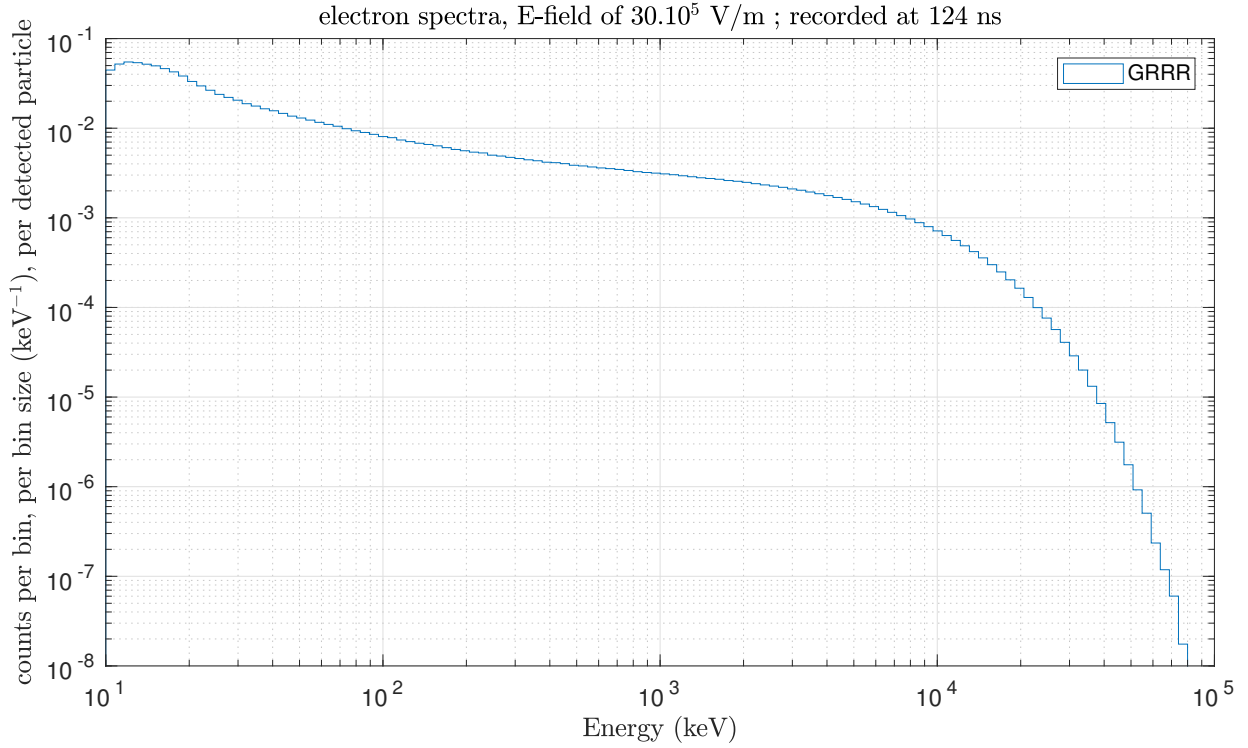


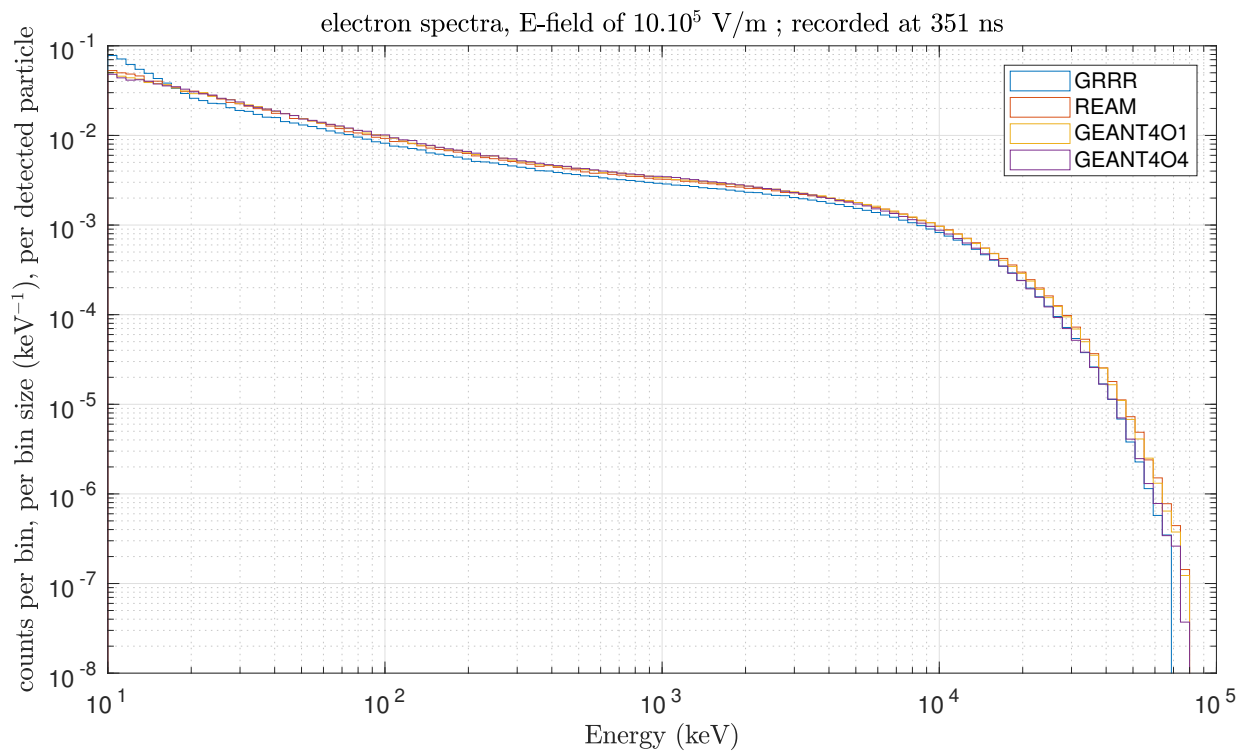
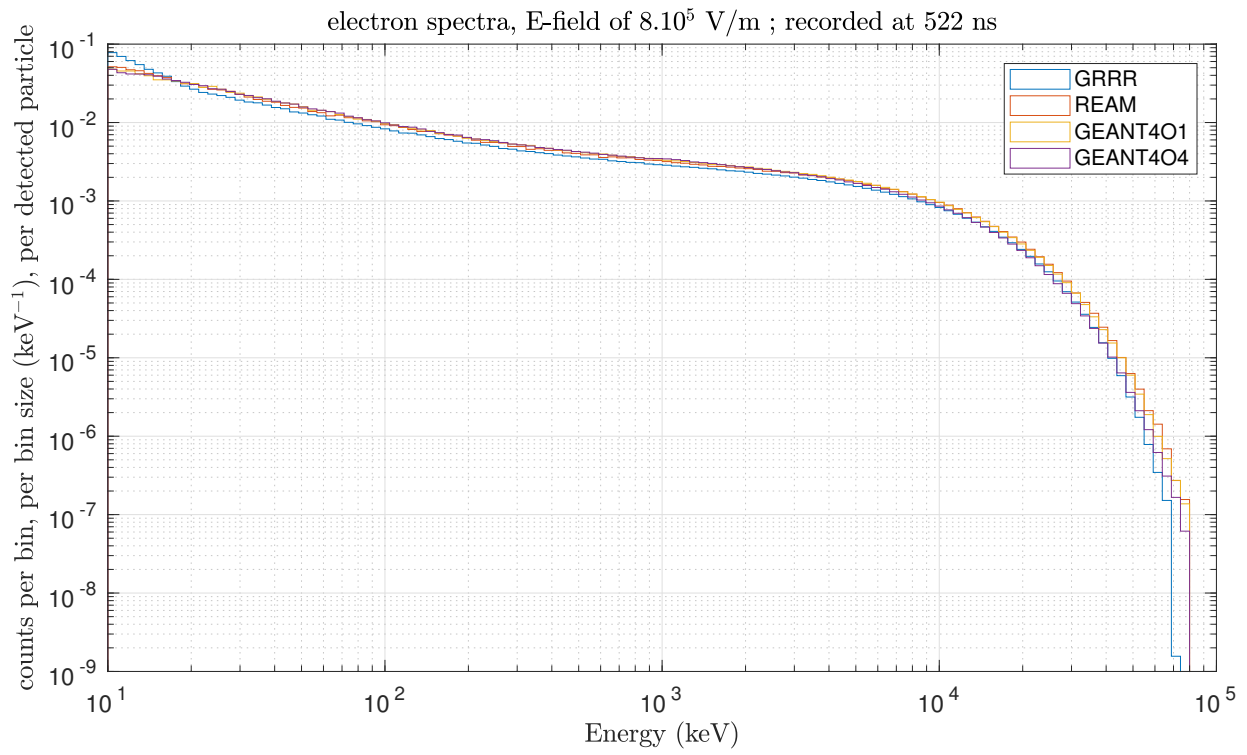
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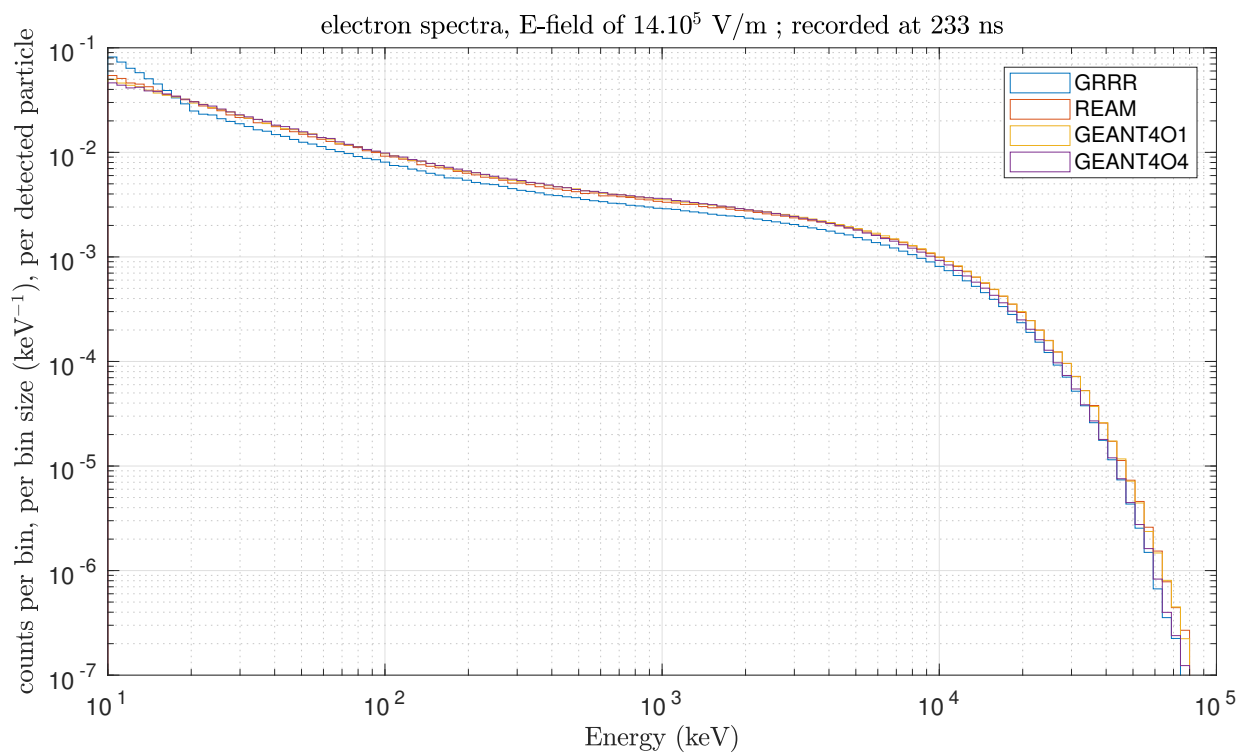
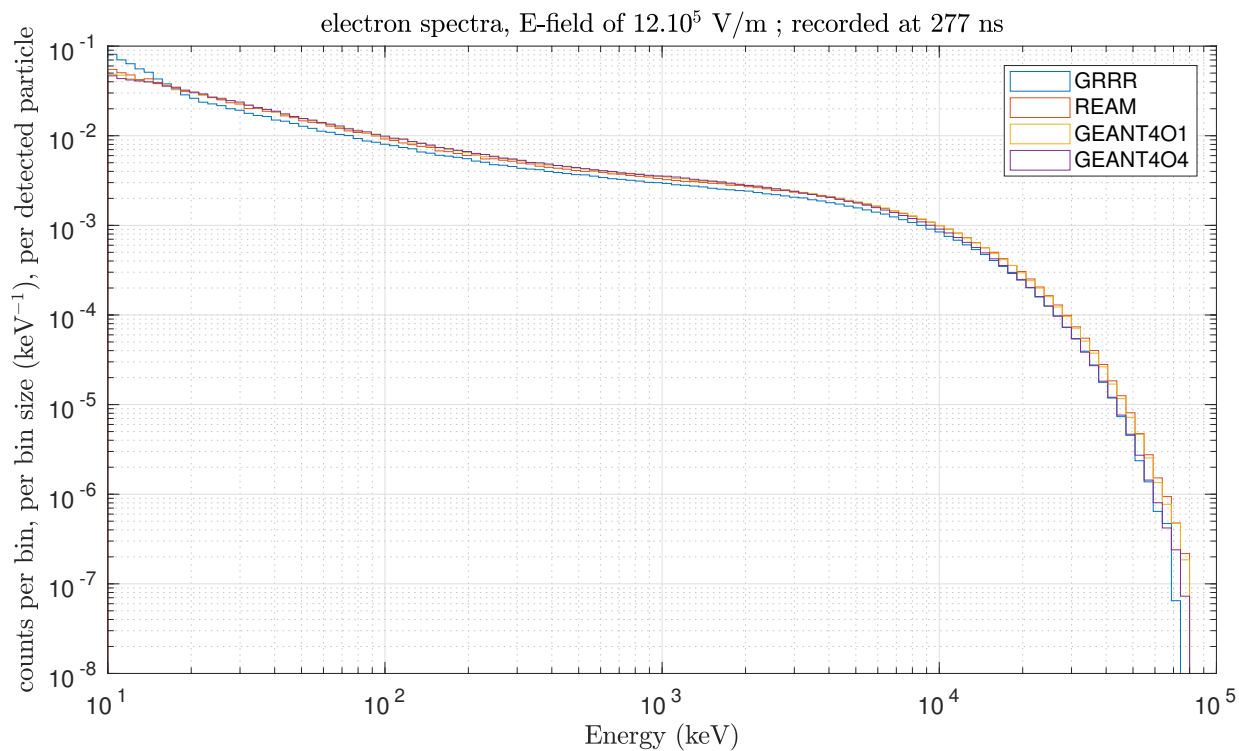


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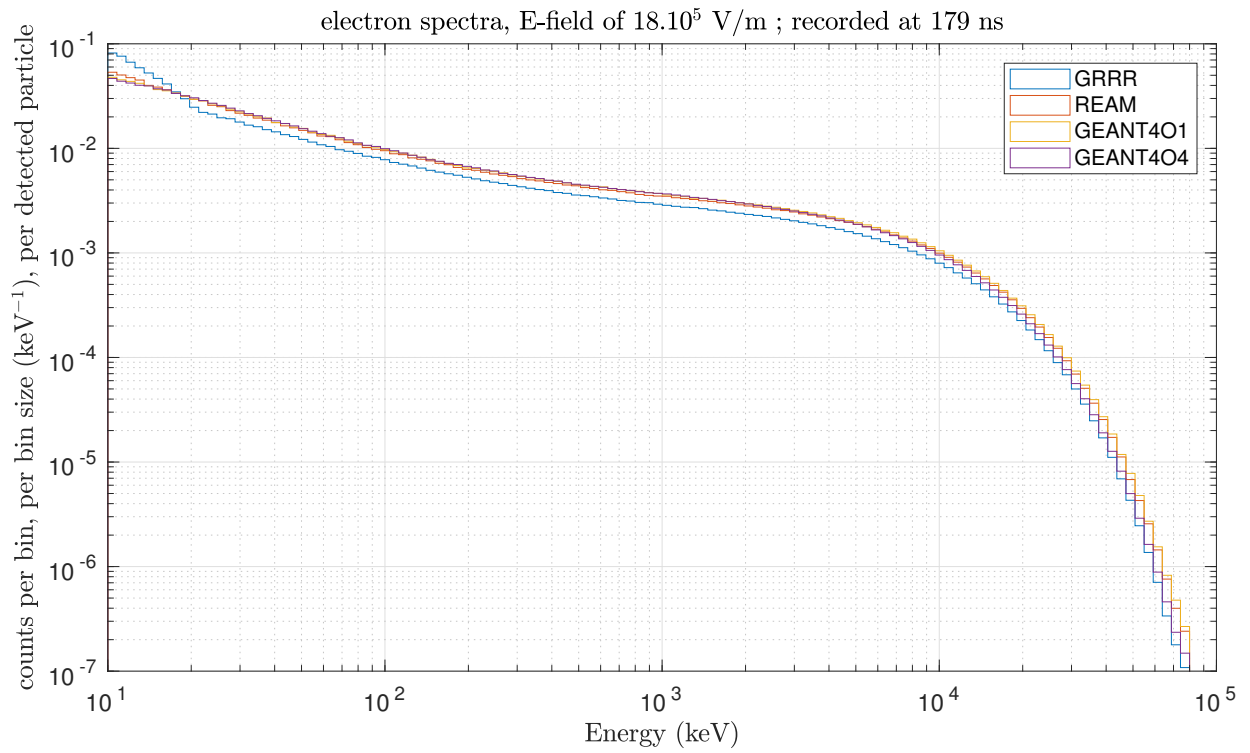
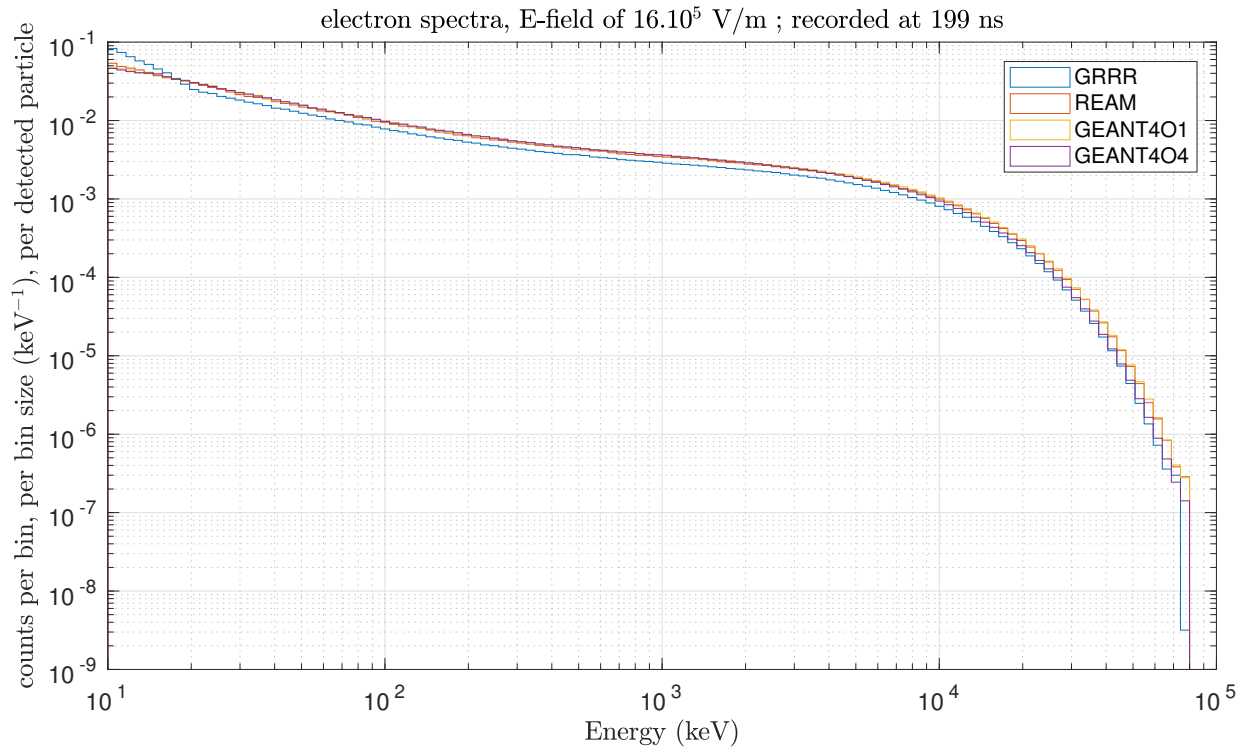
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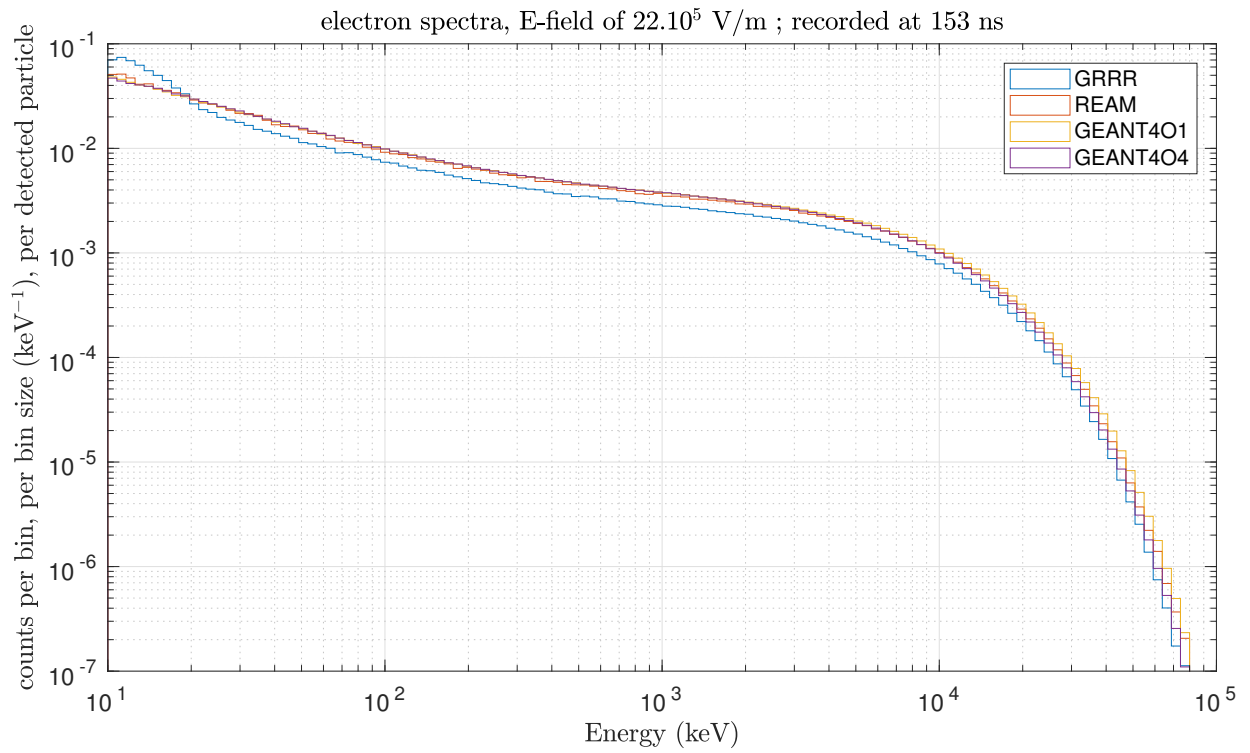
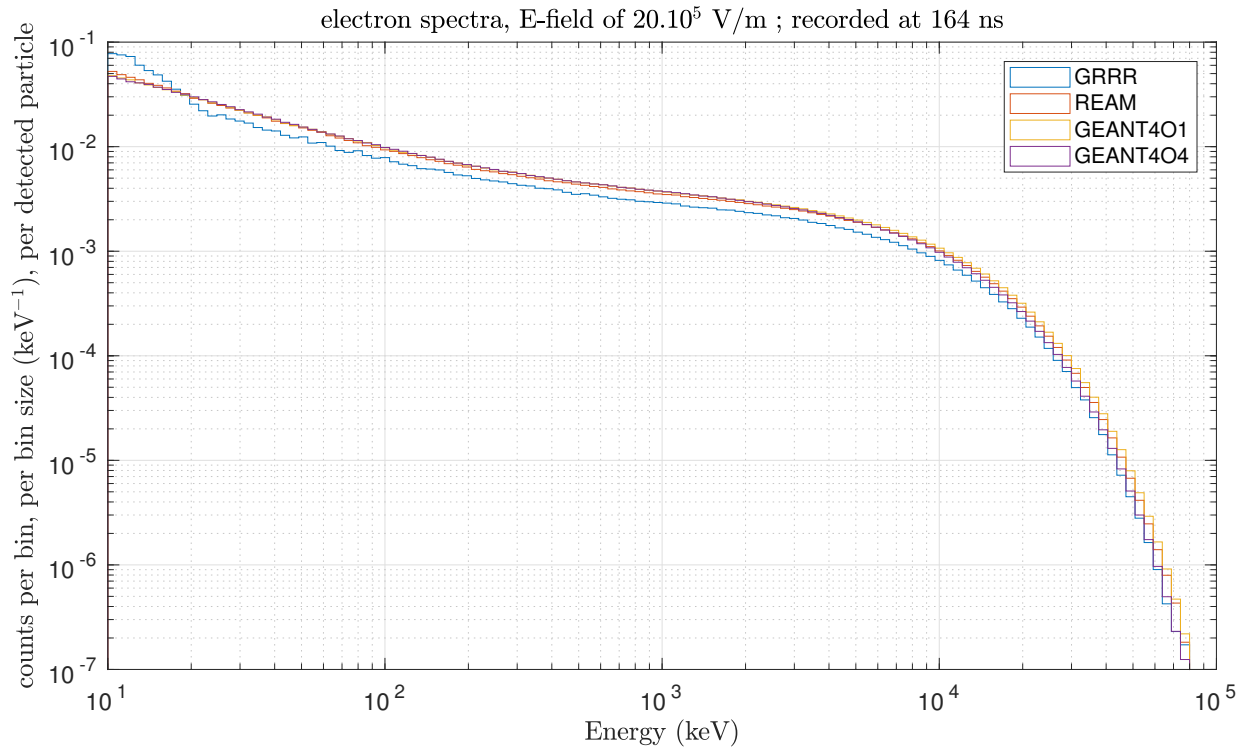


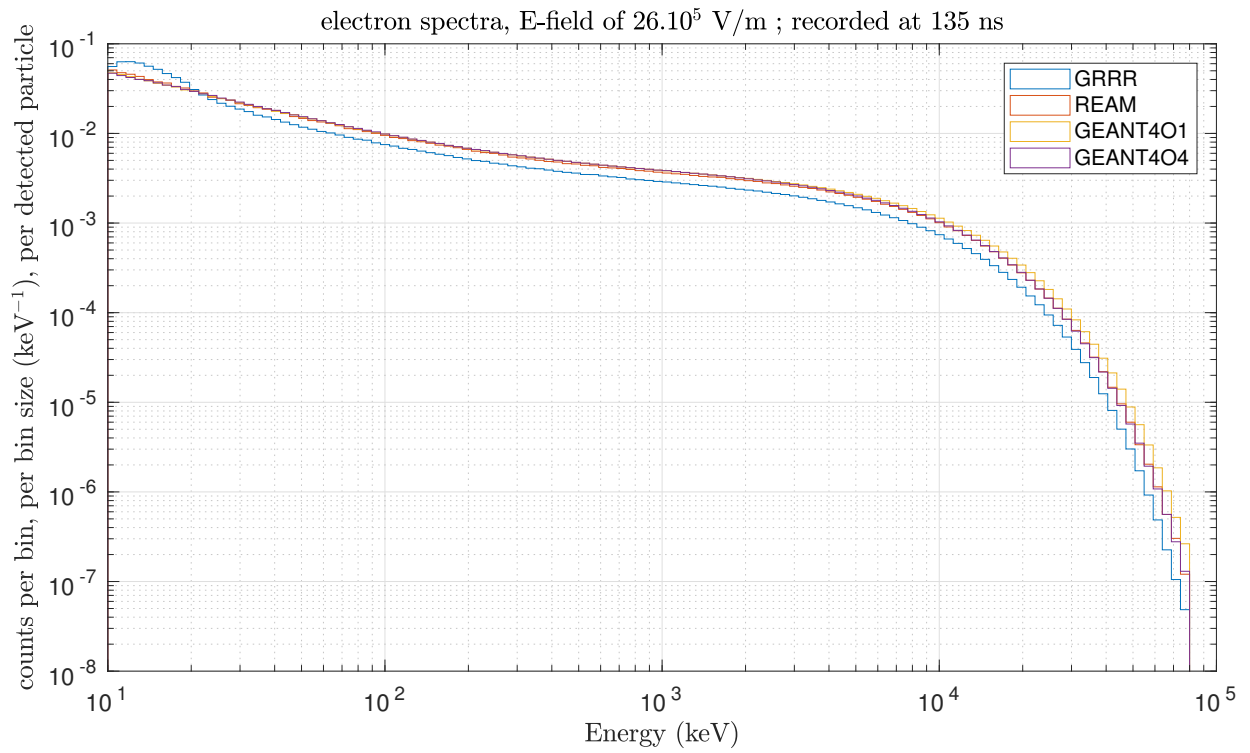
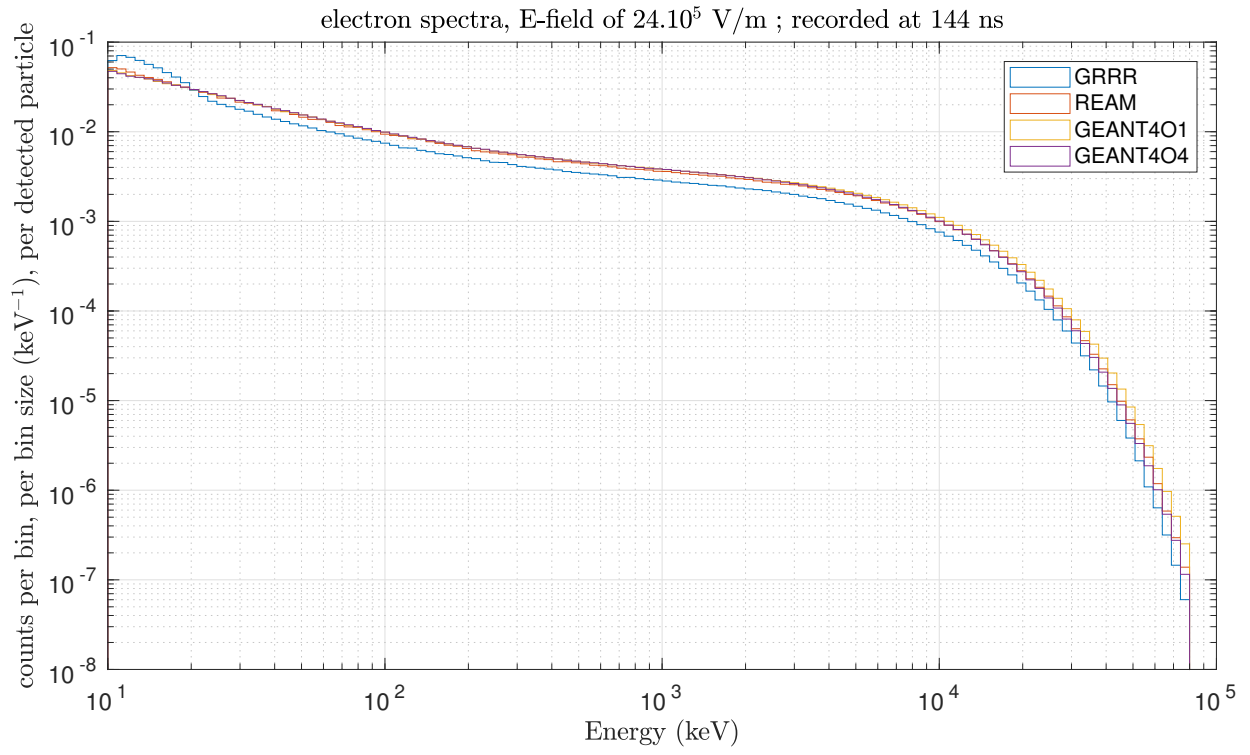


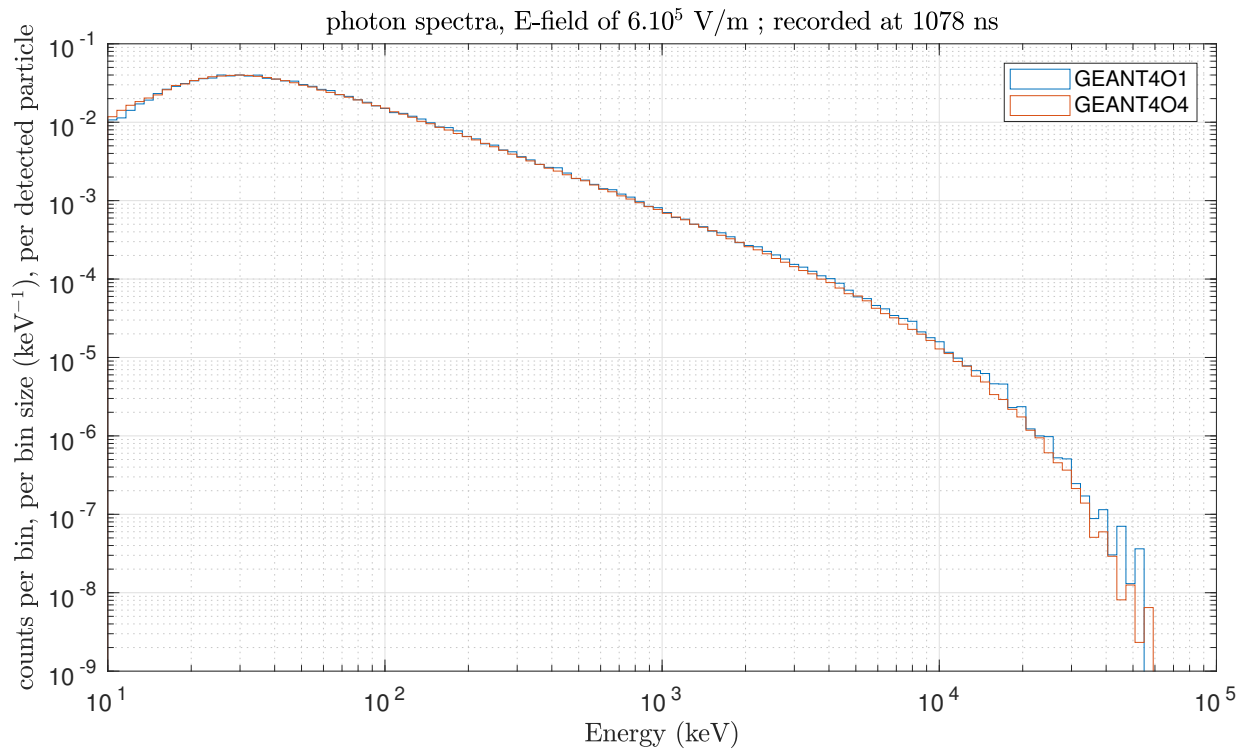
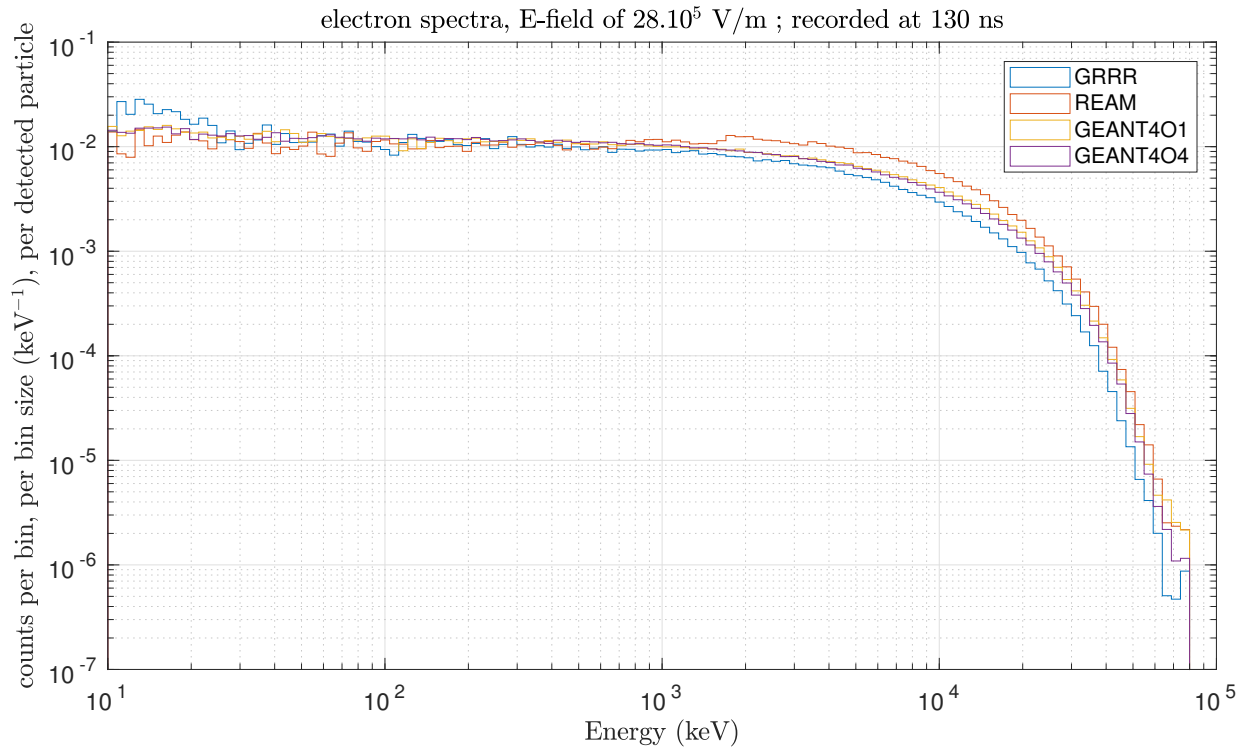


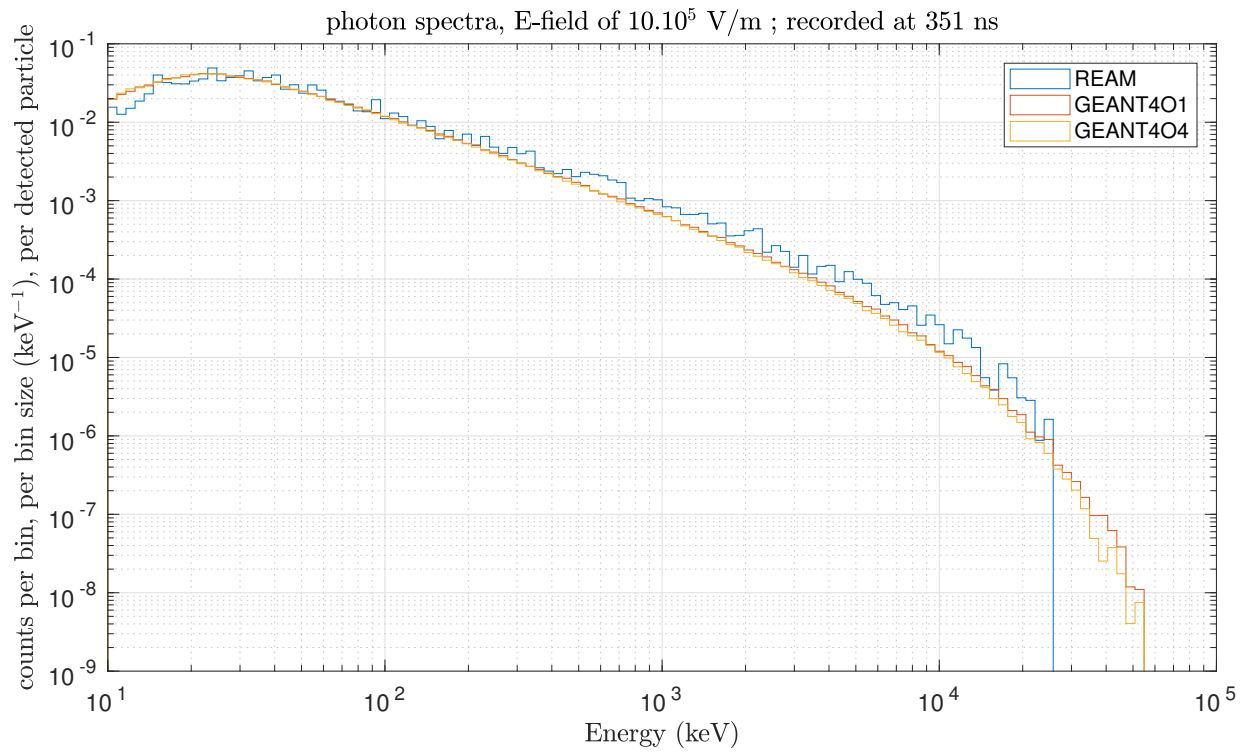
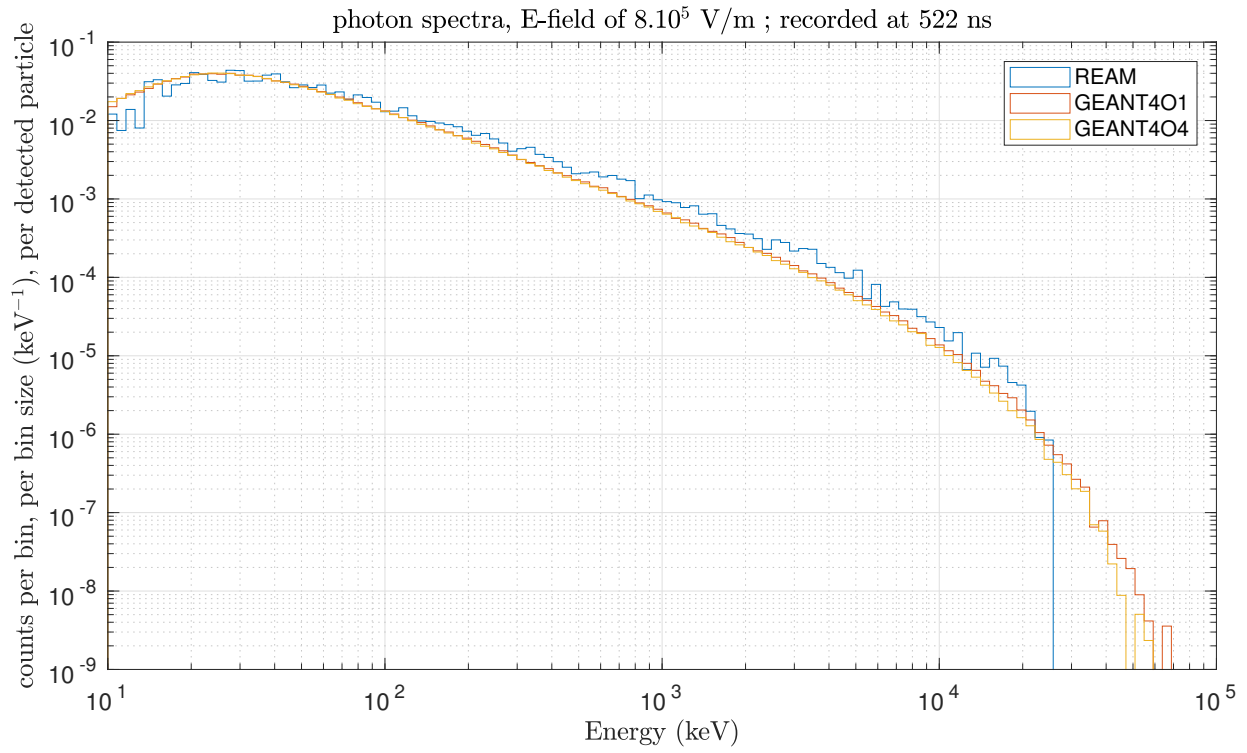


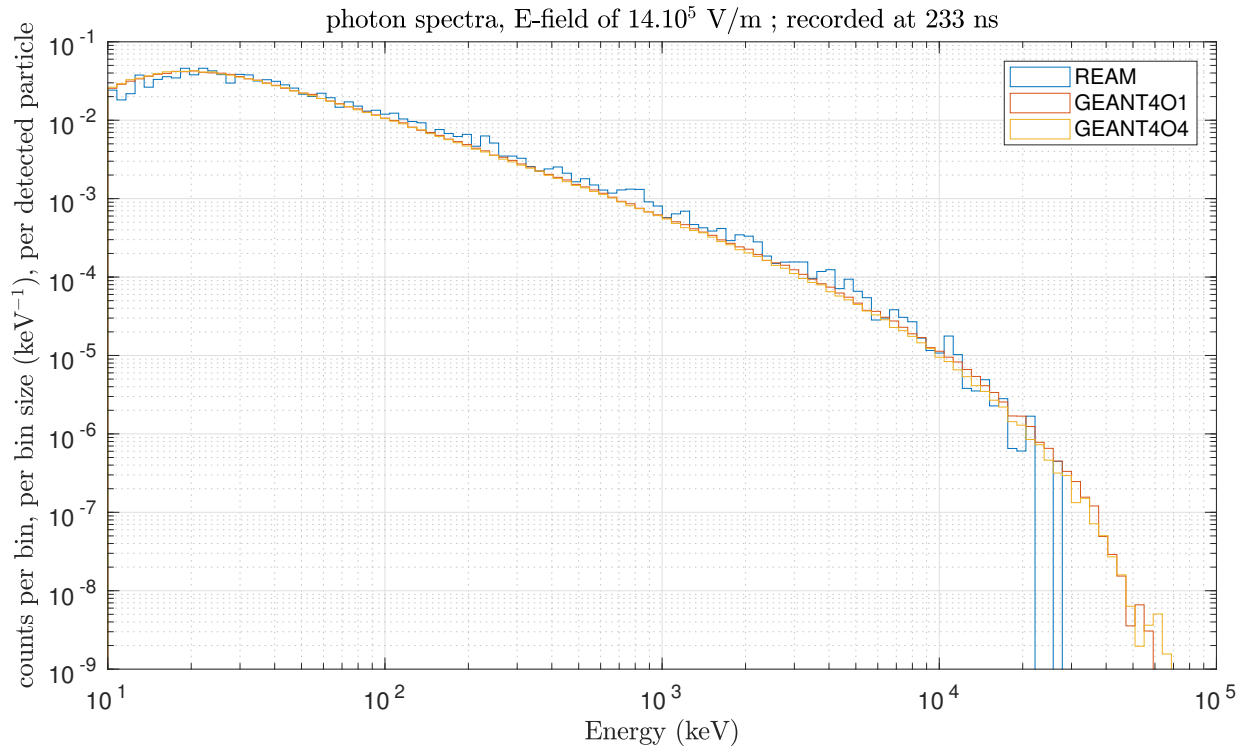
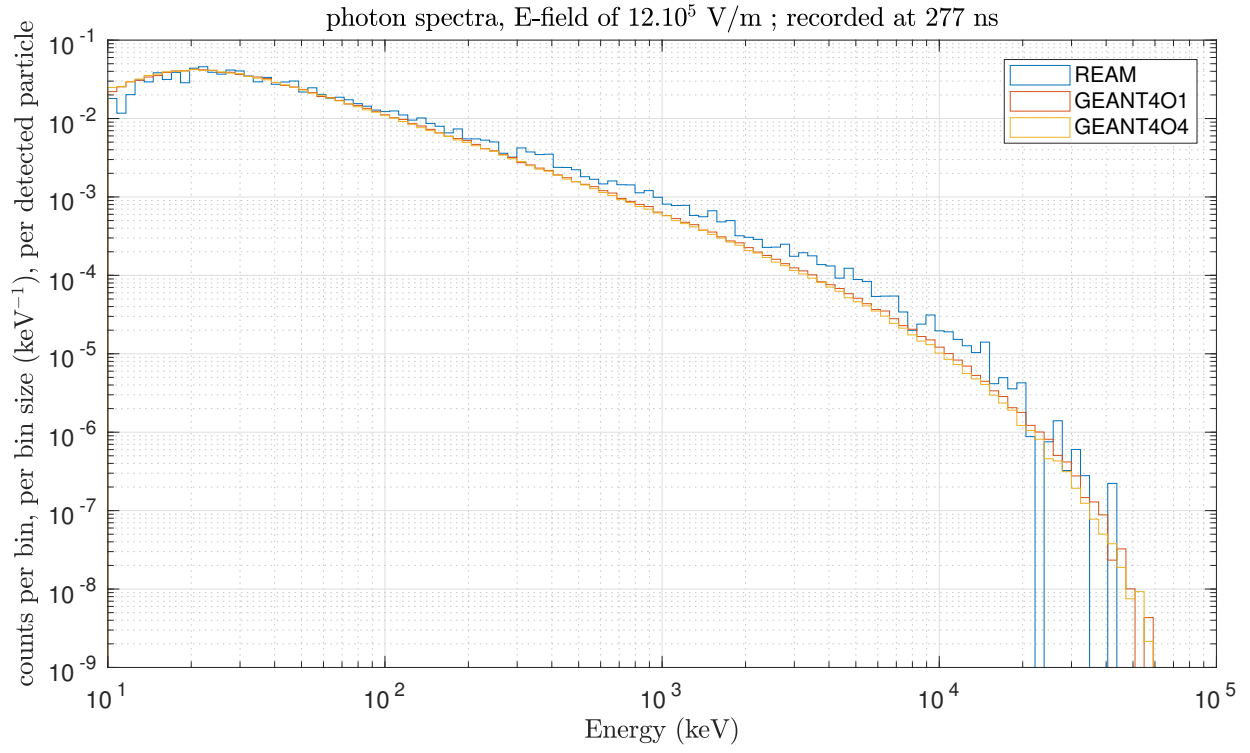


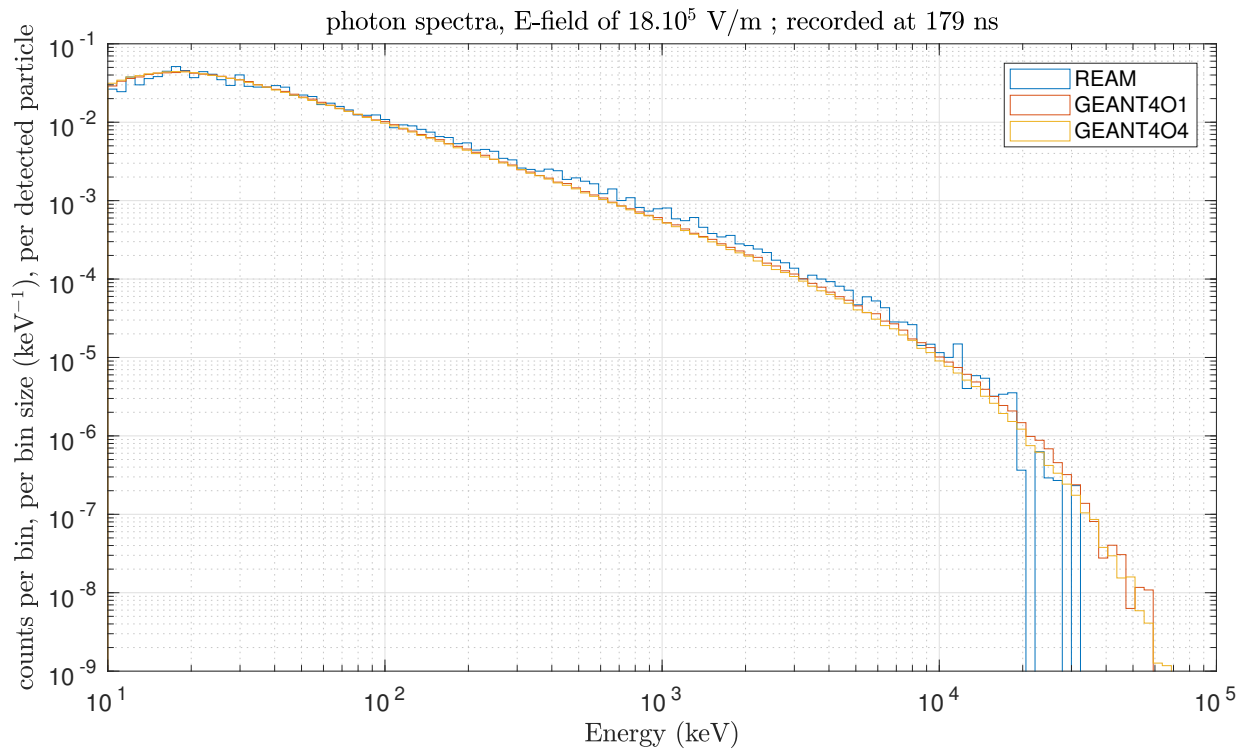
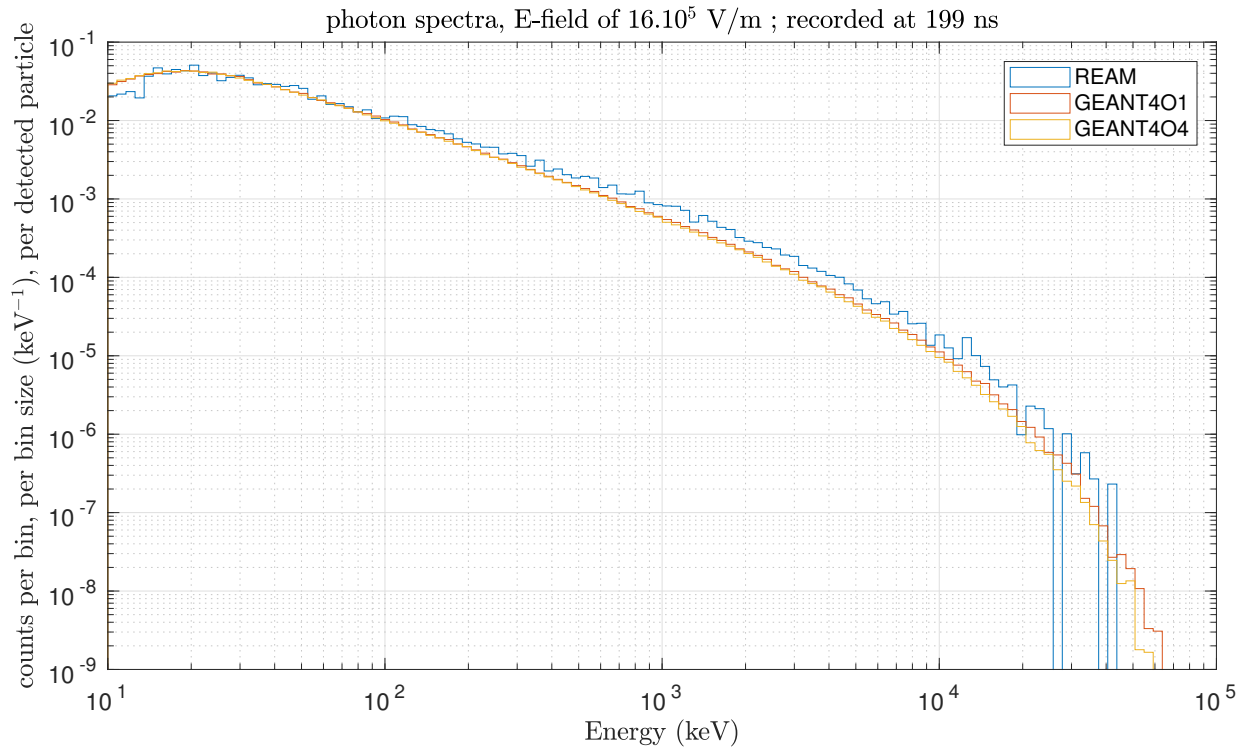


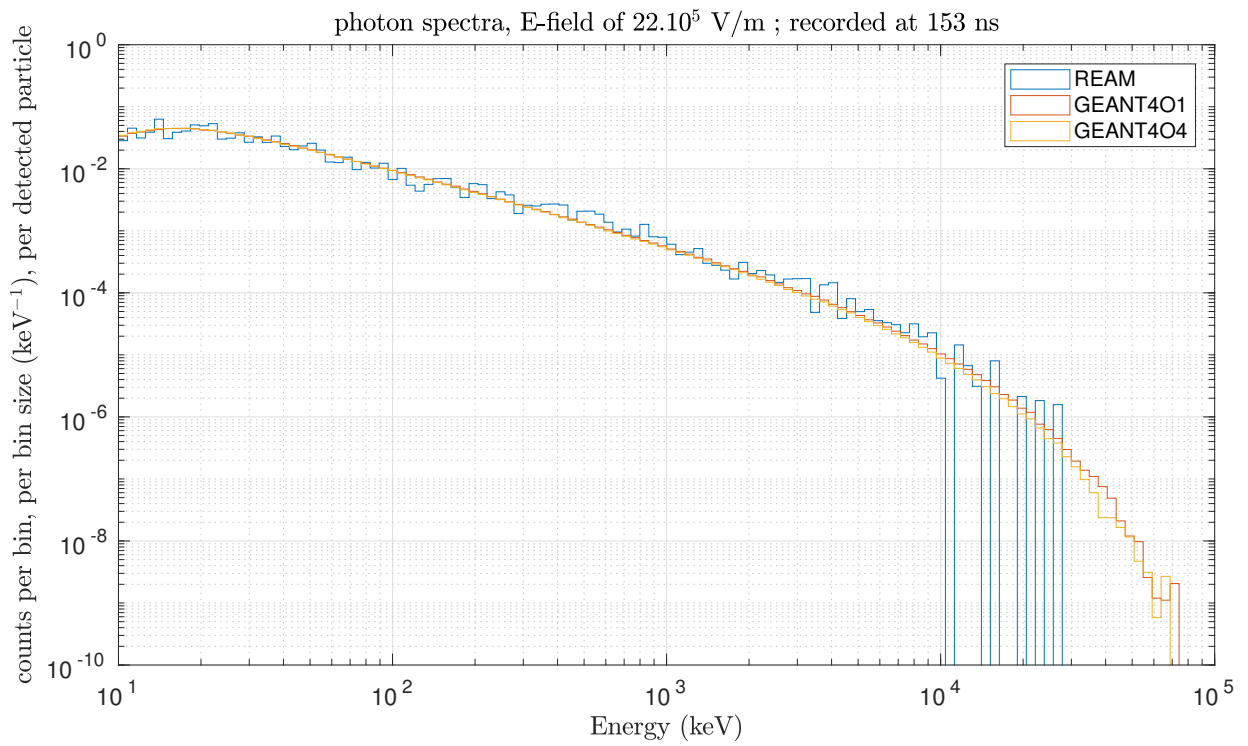
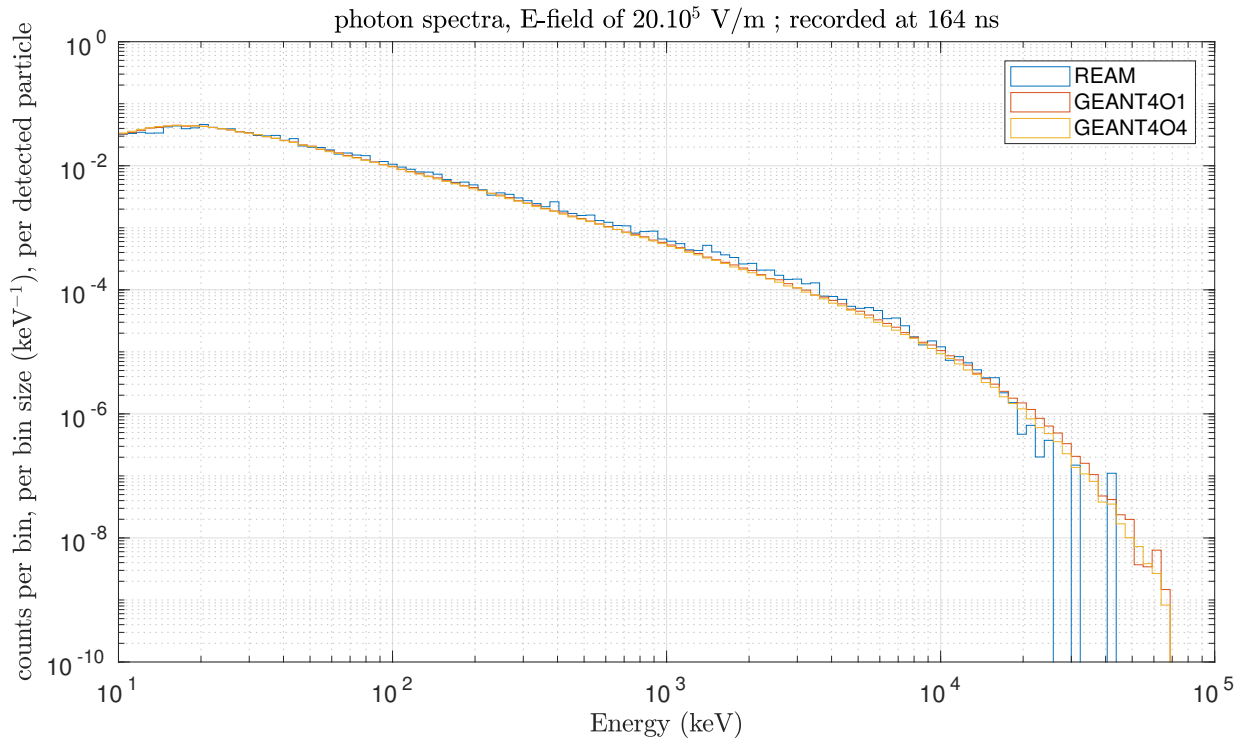




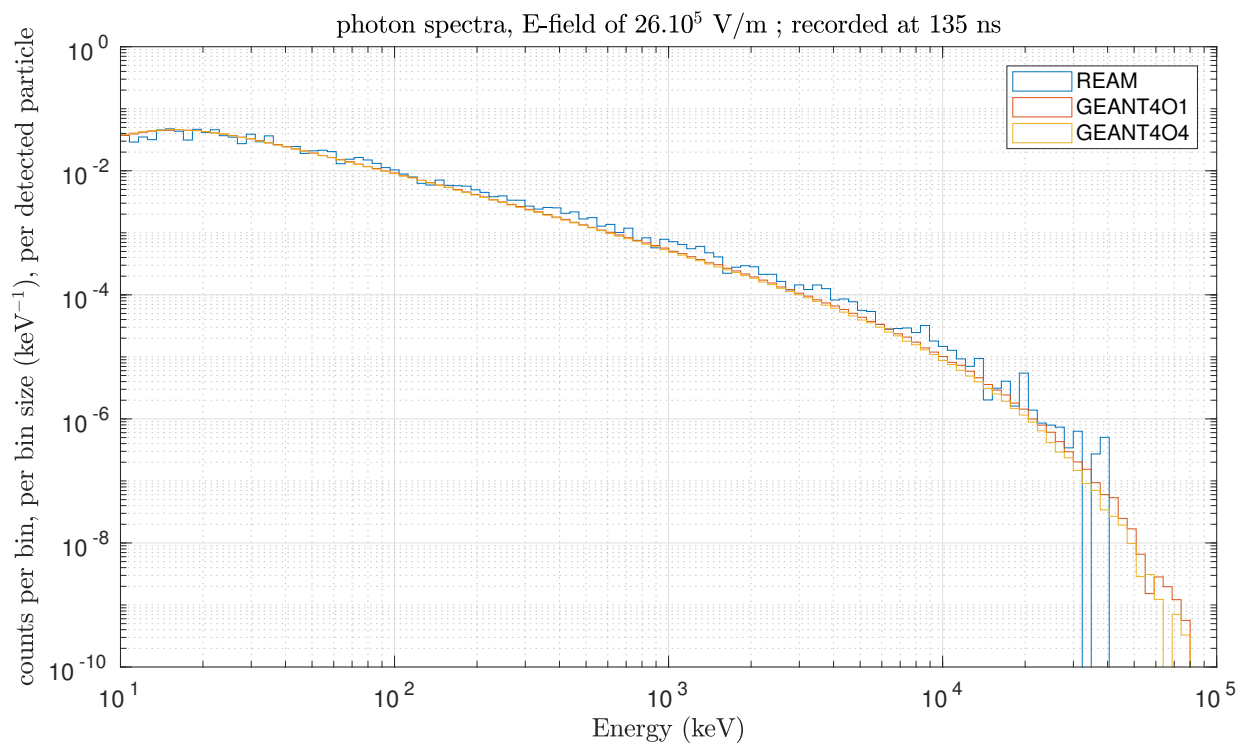
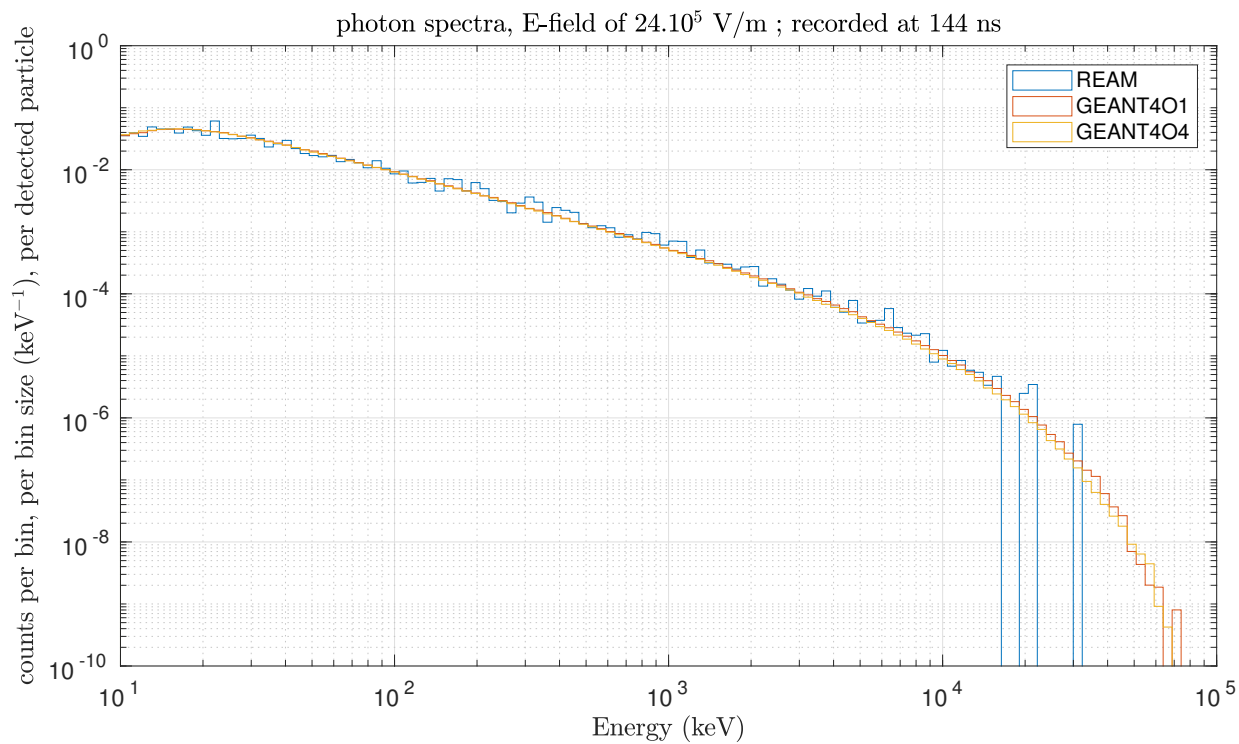


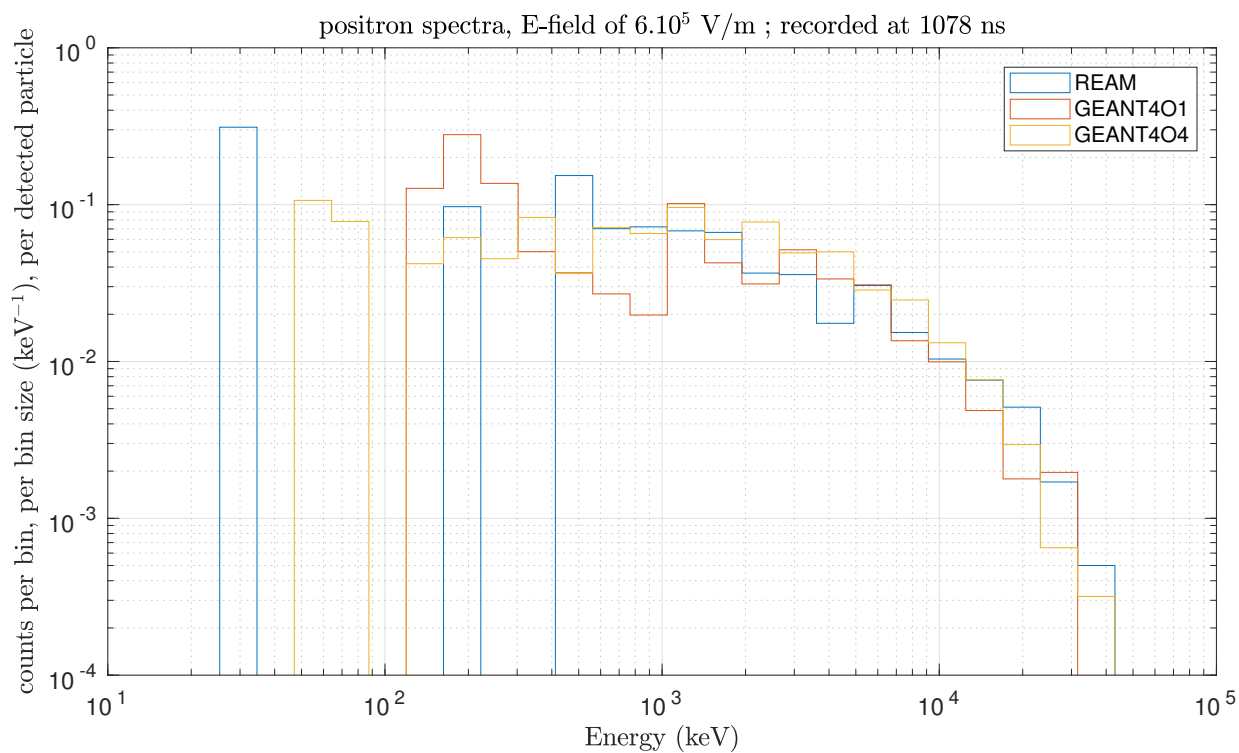
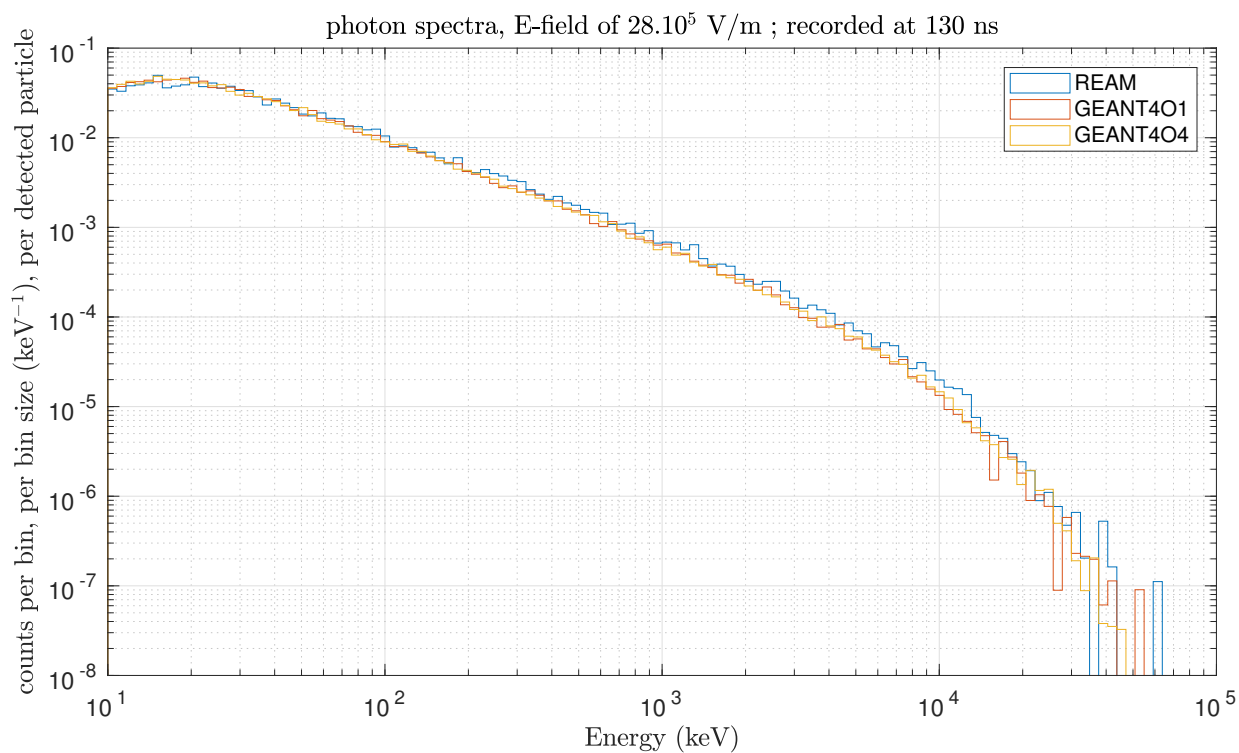


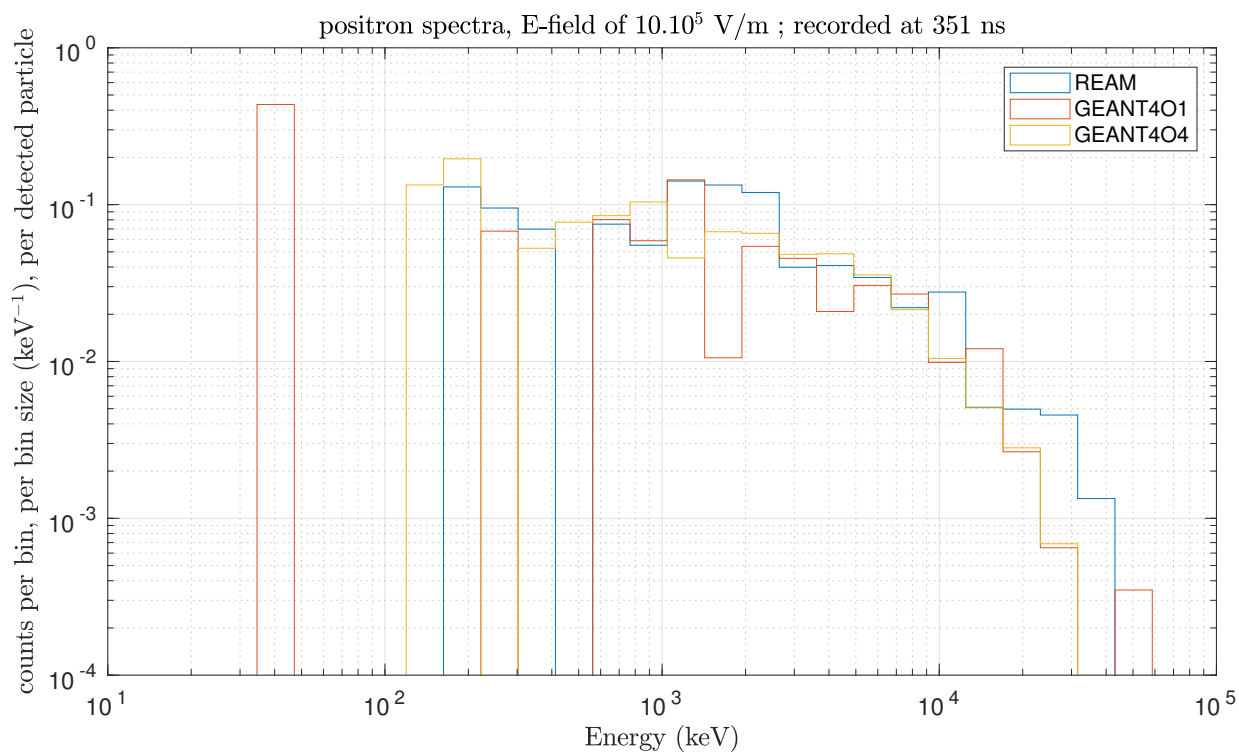
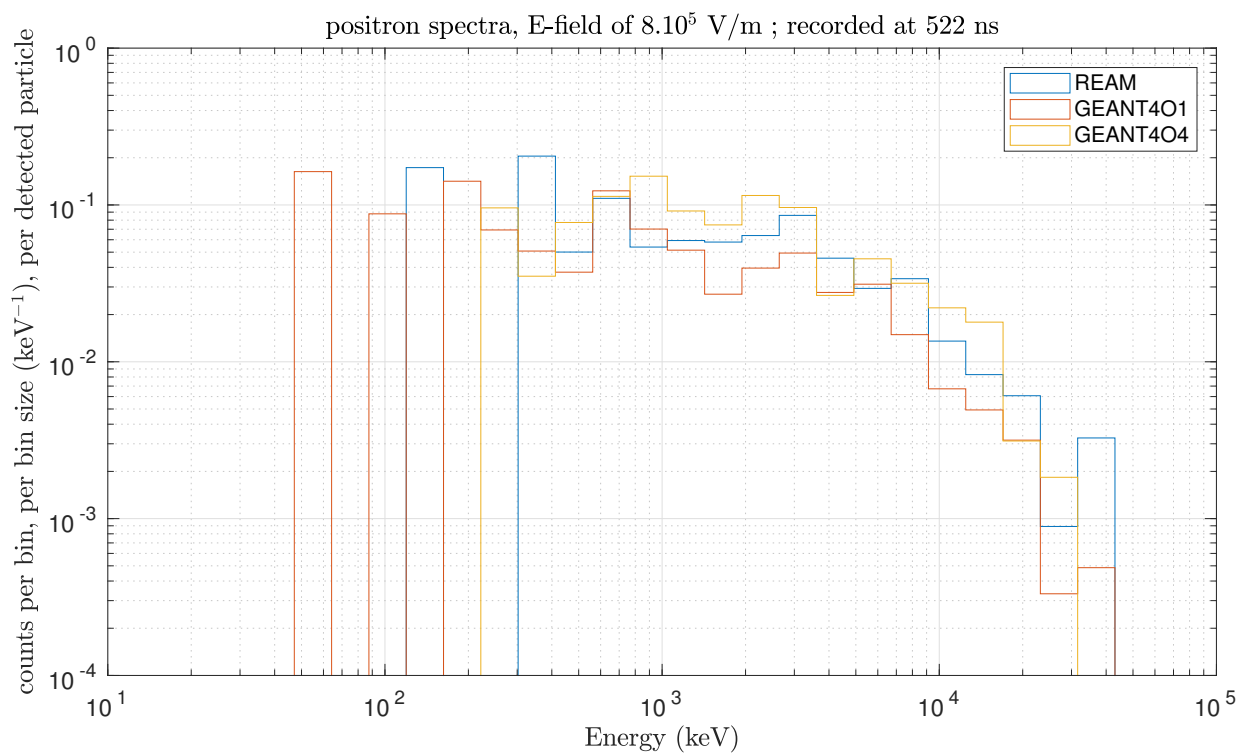


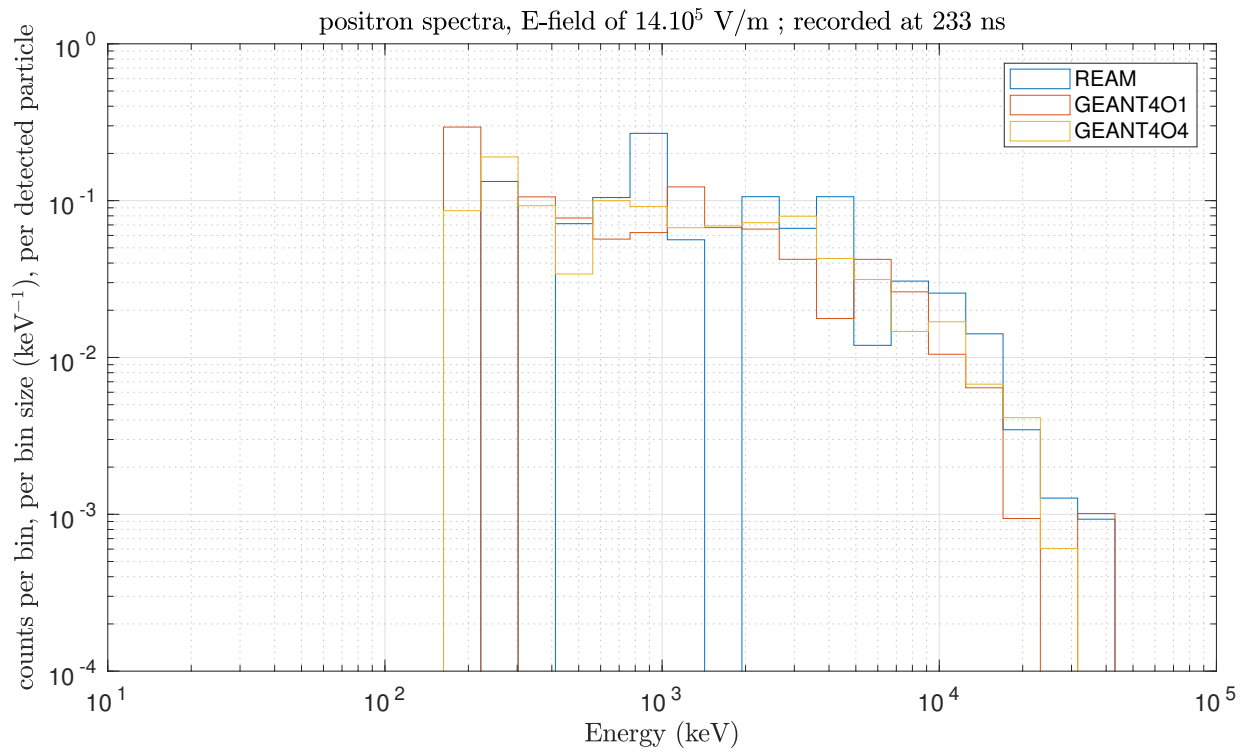
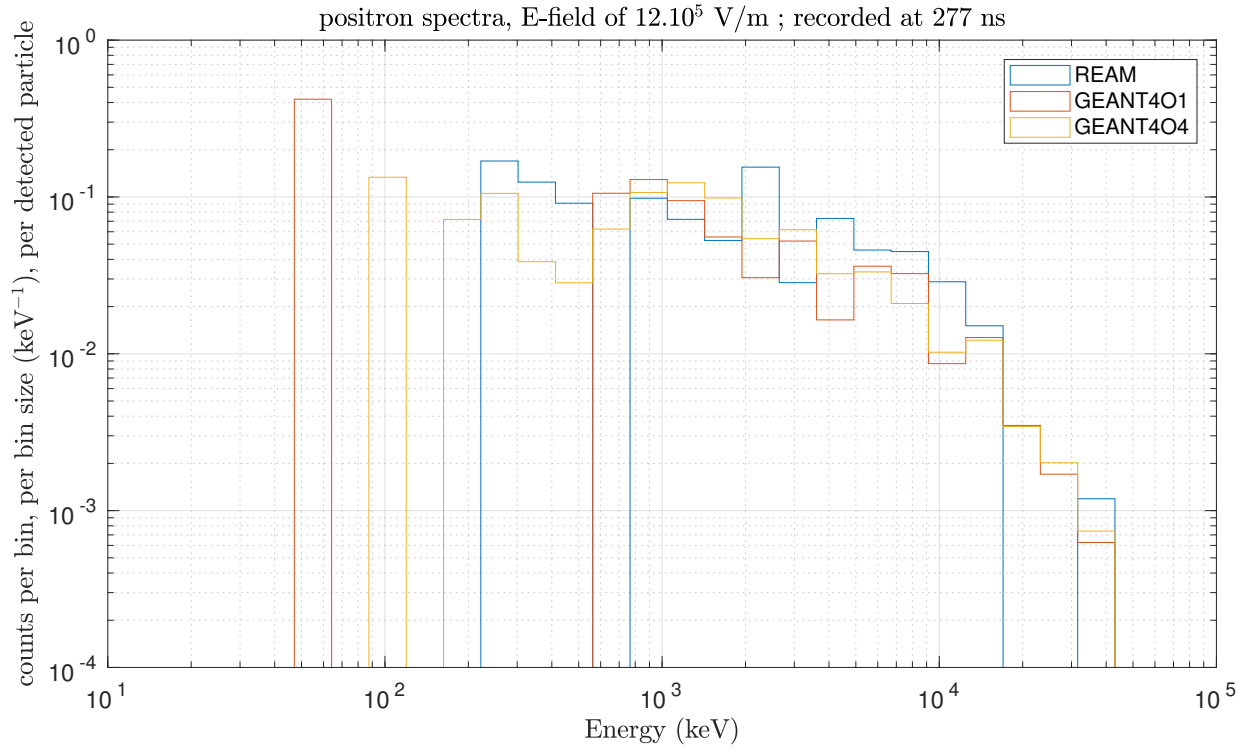


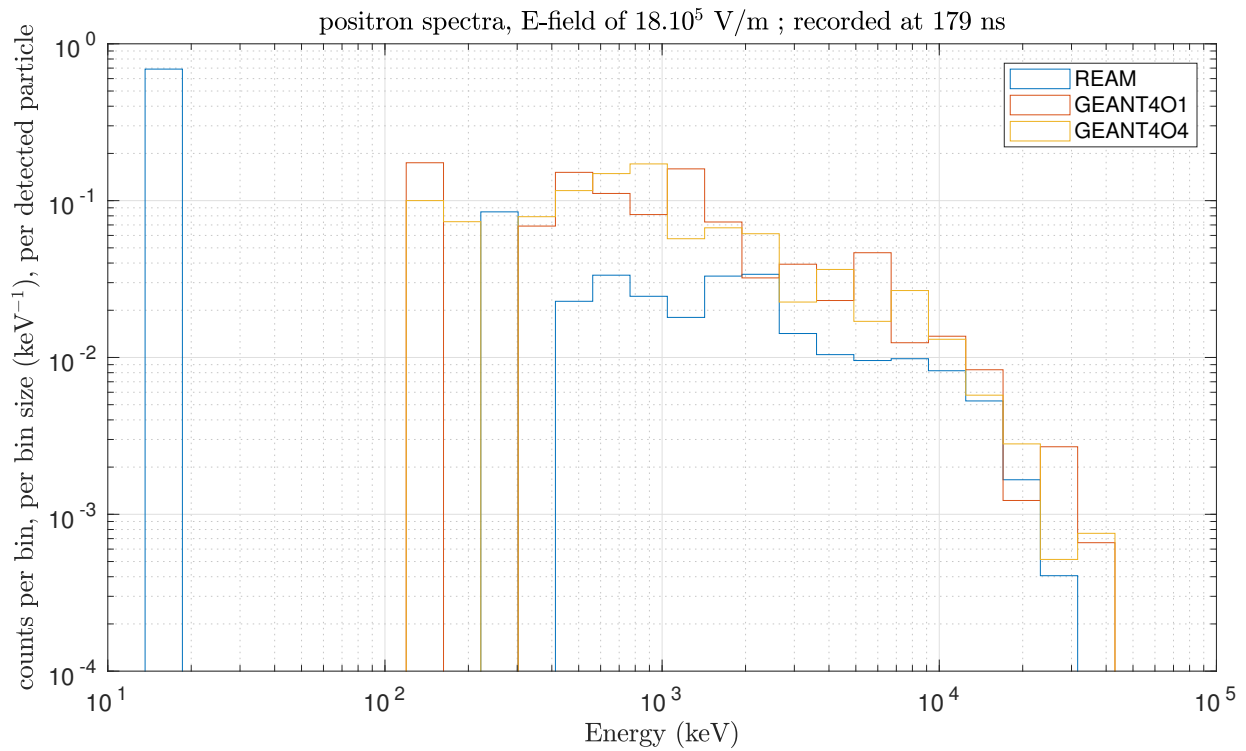
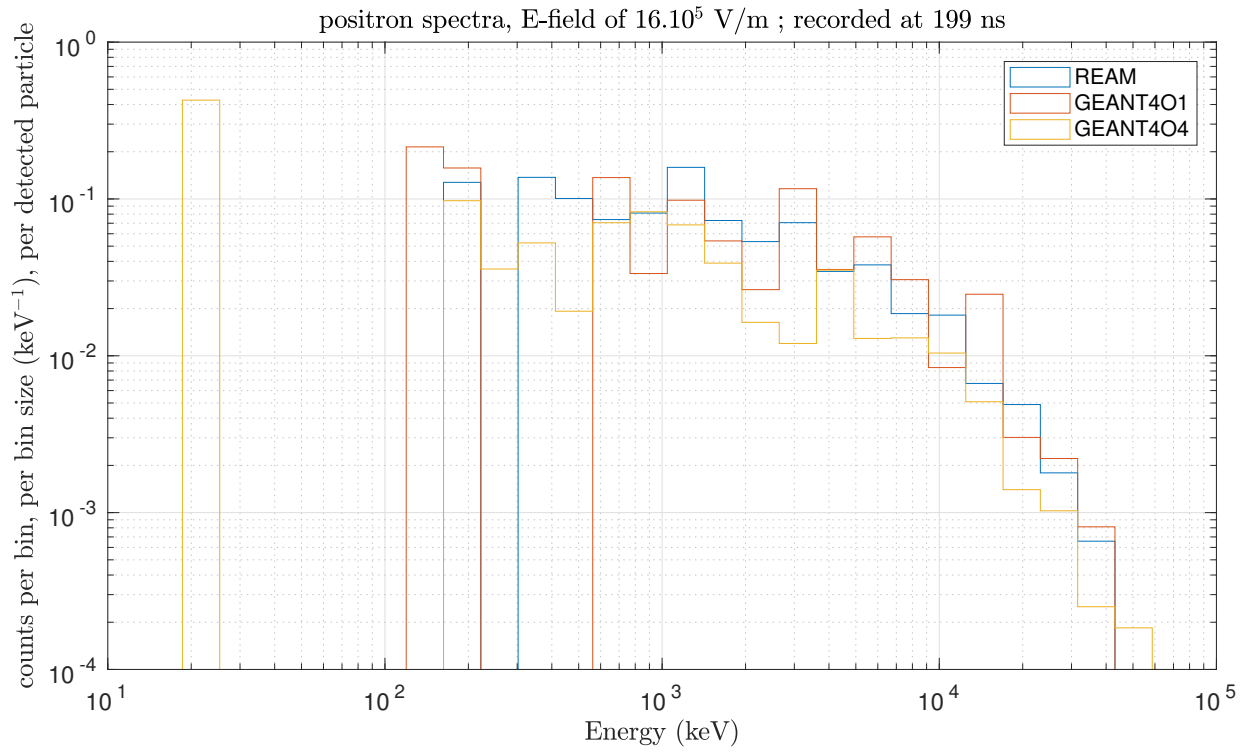


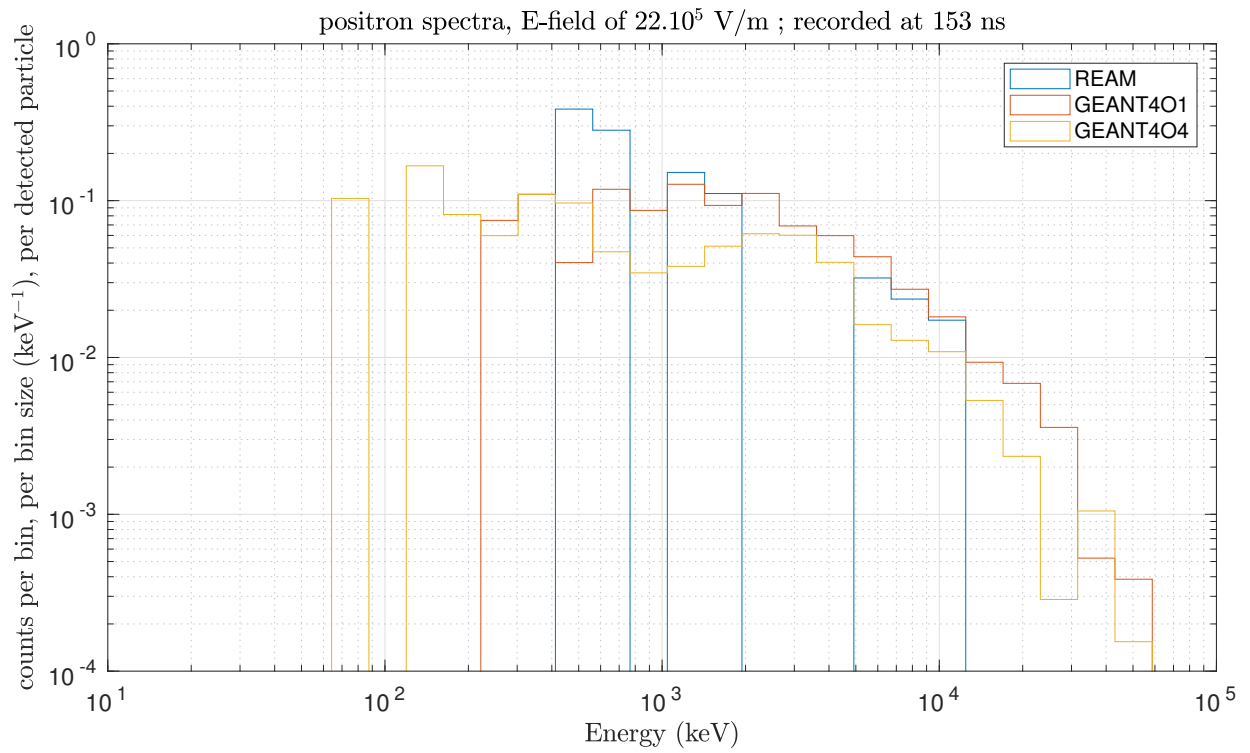
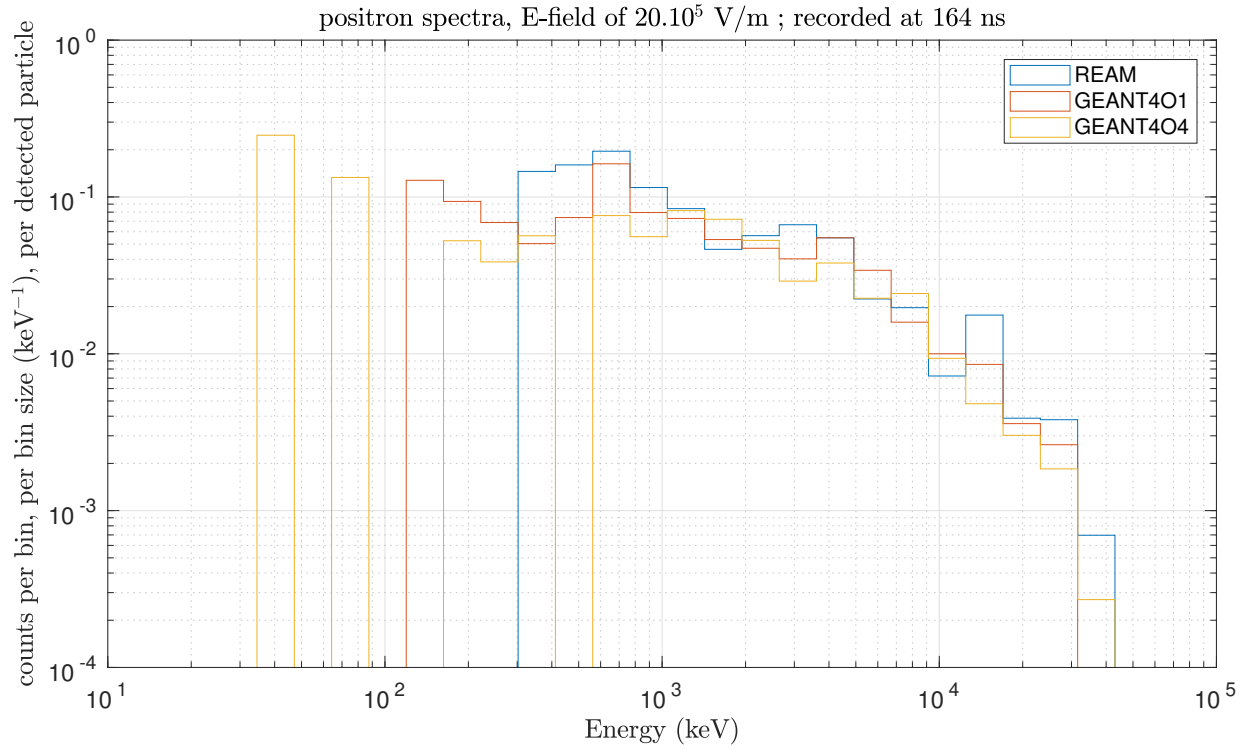


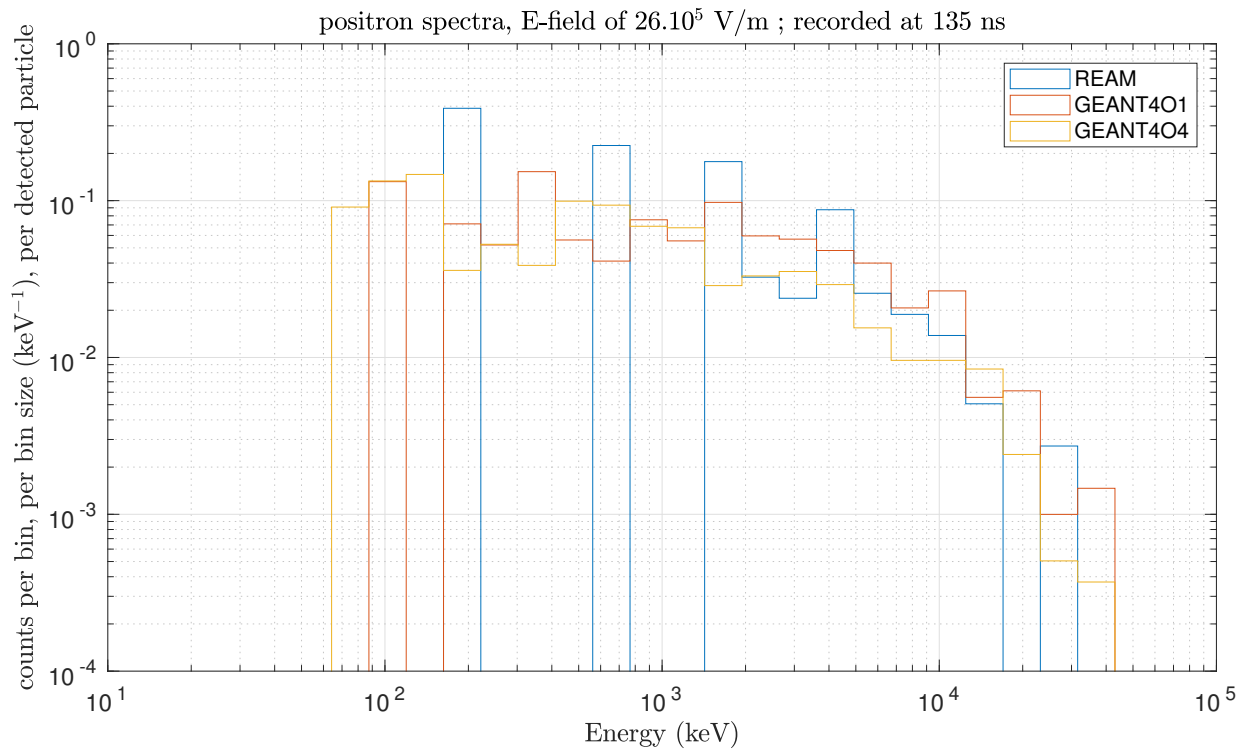
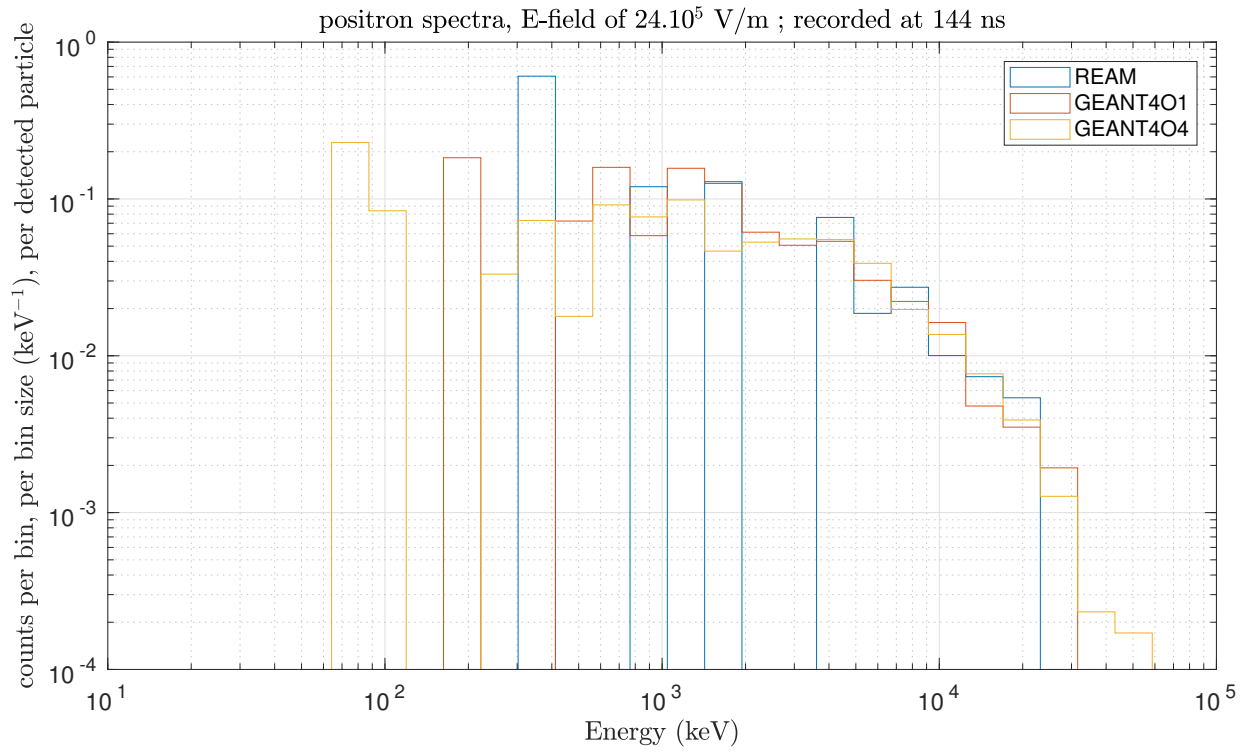


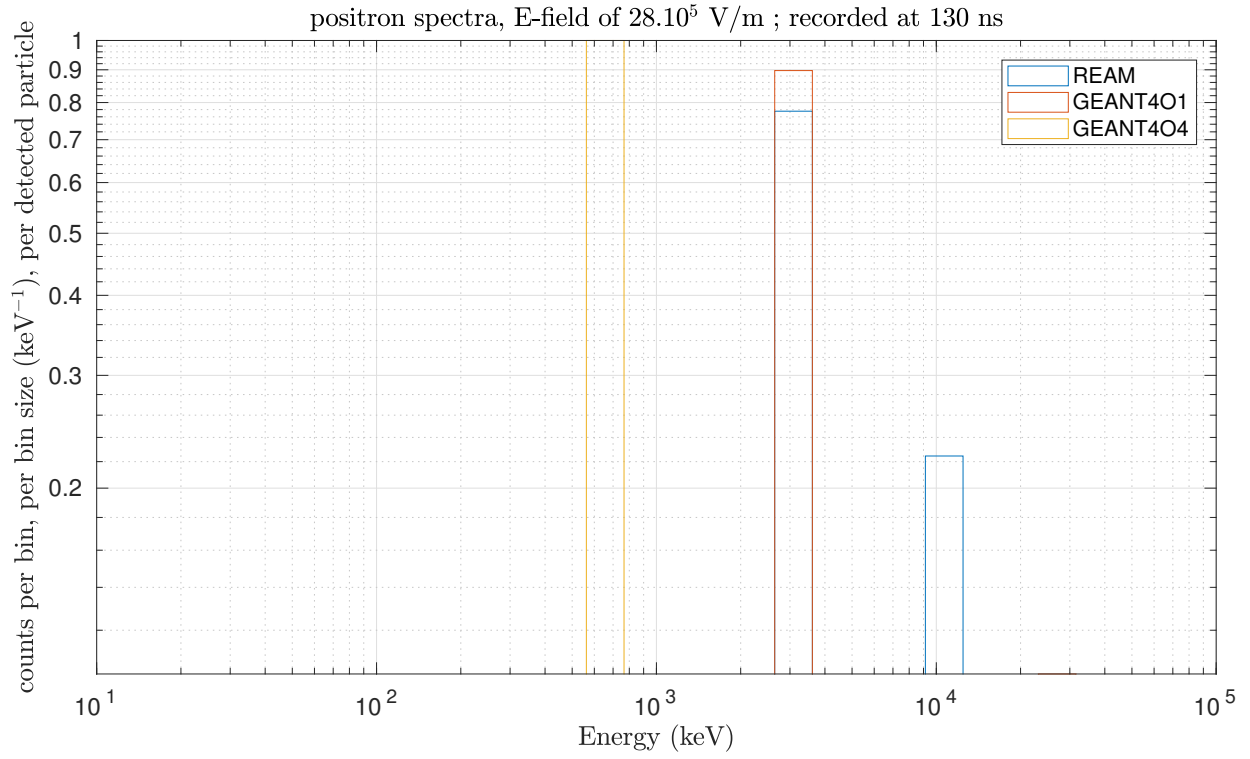




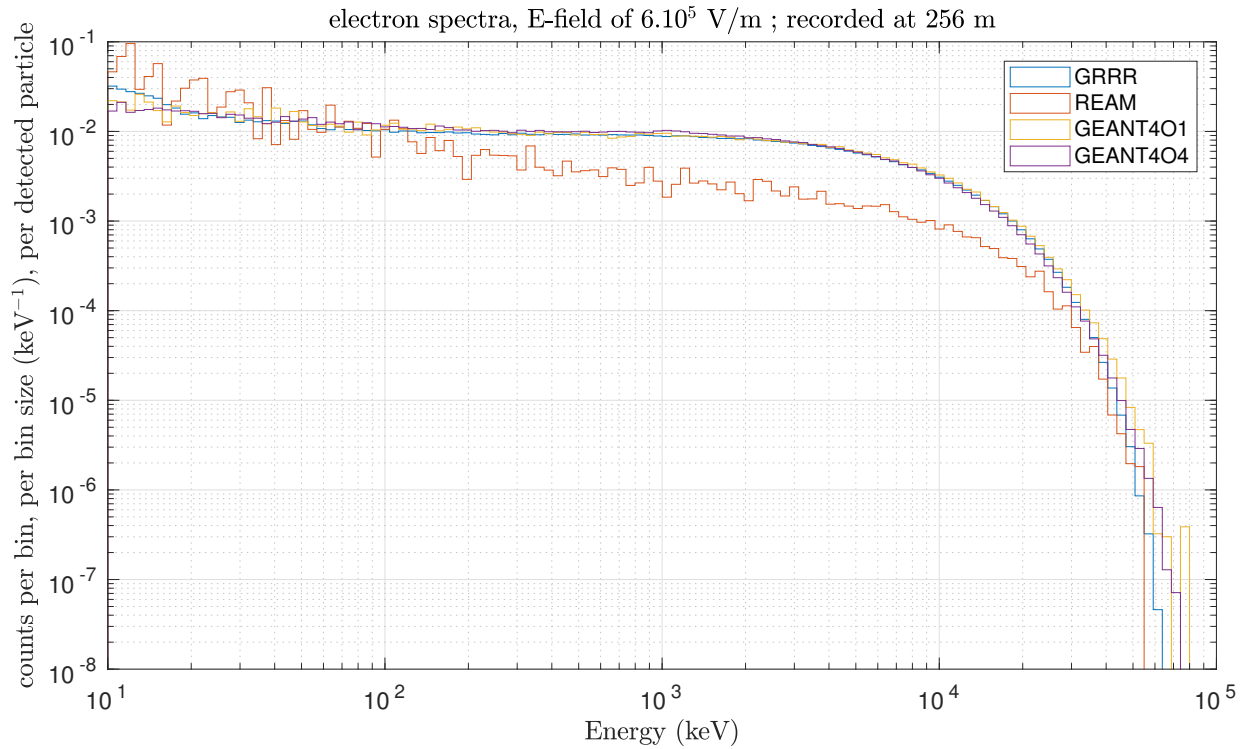




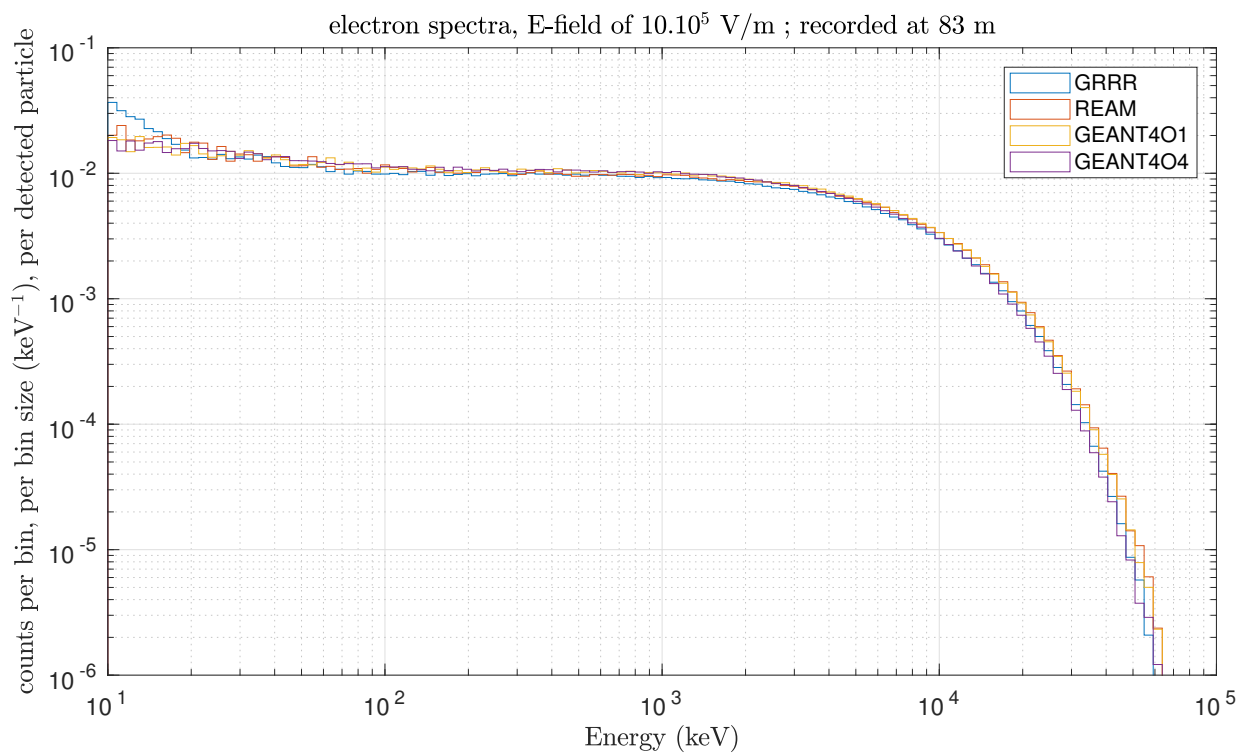
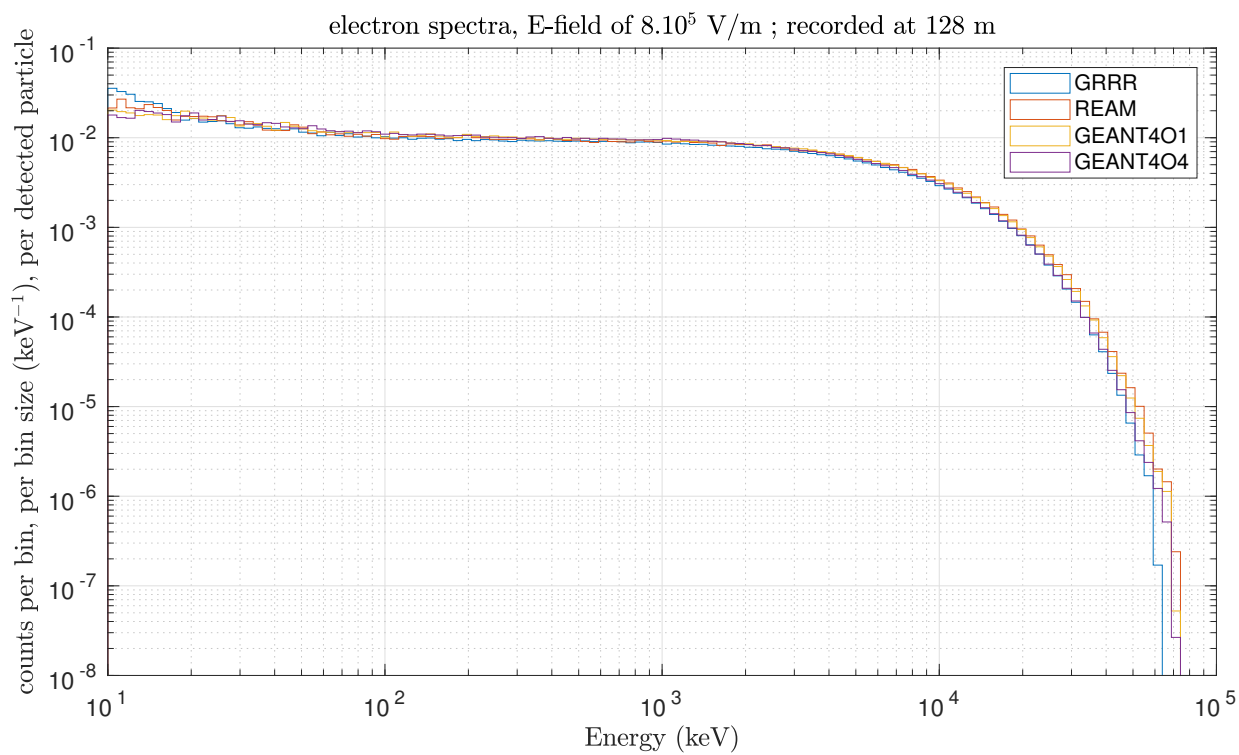


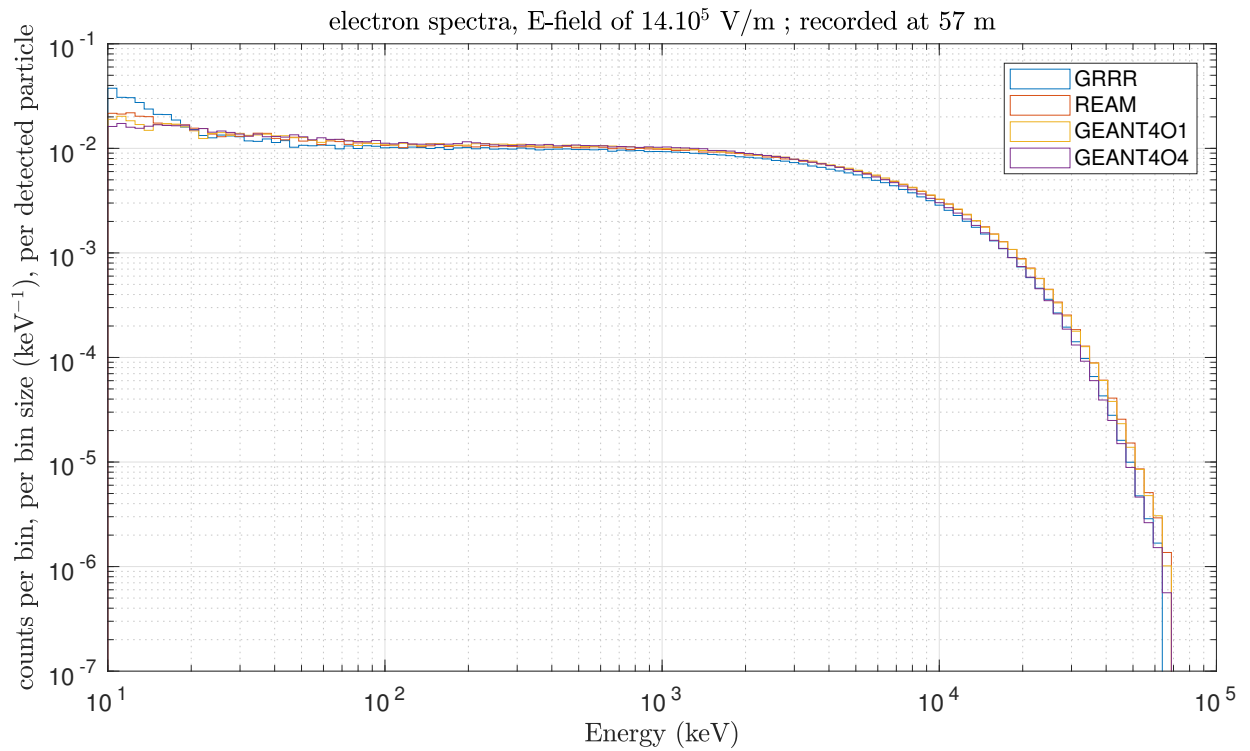
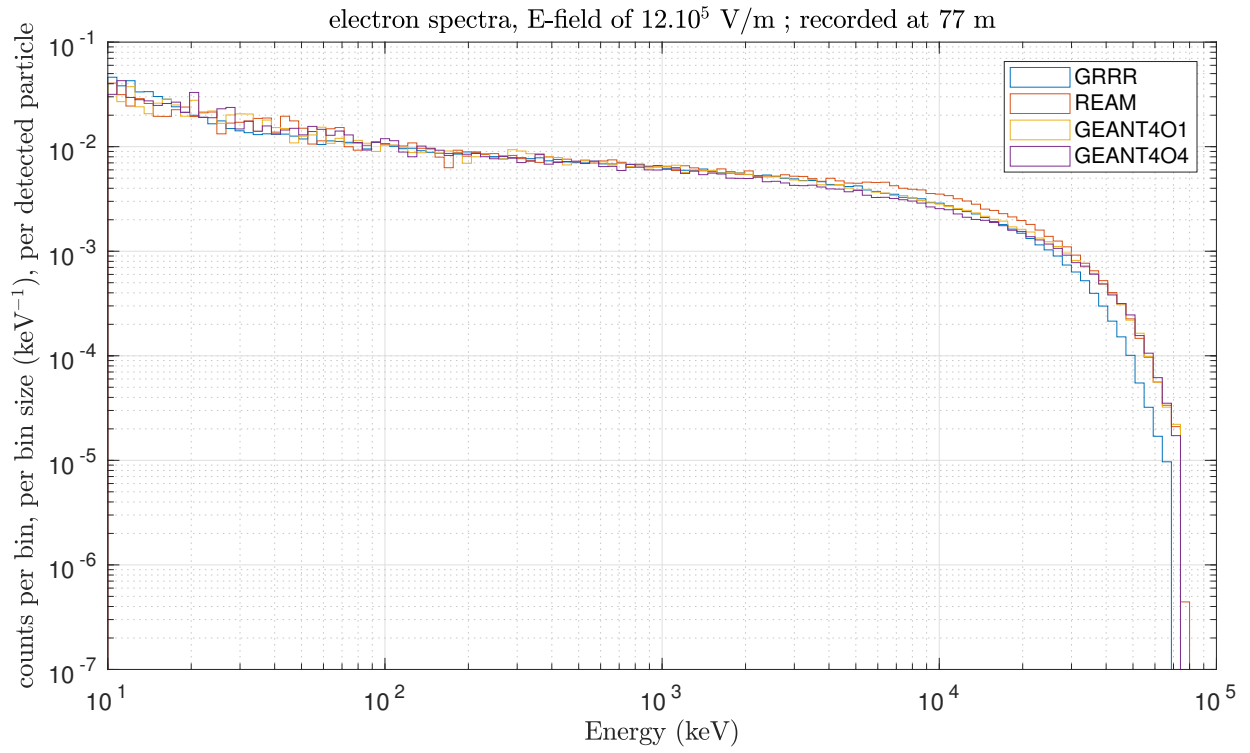


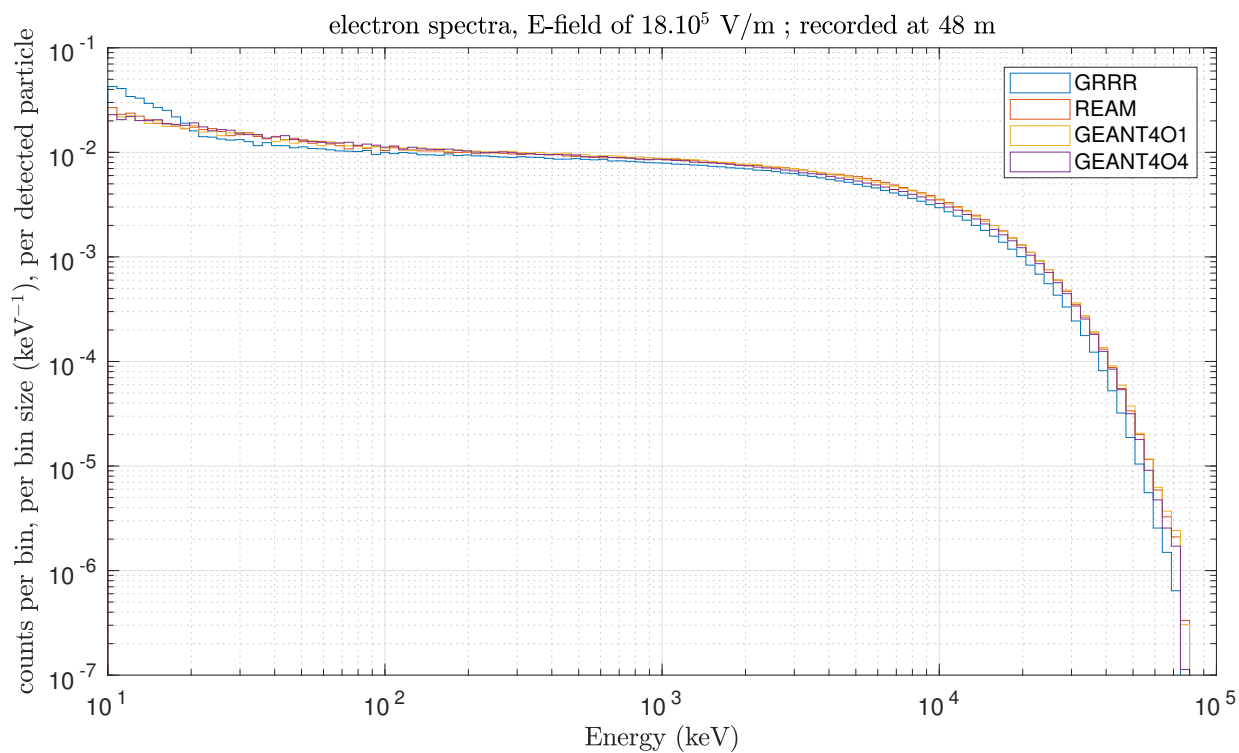
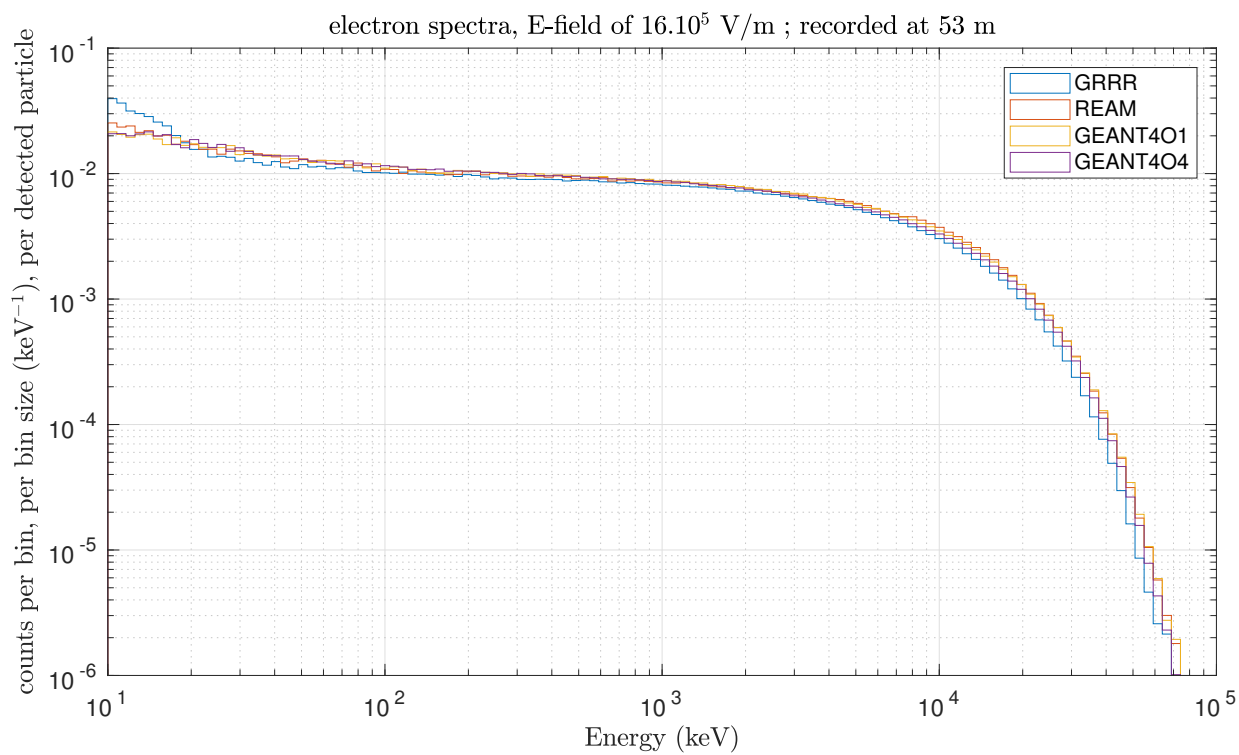
## 6.2 Distance Record

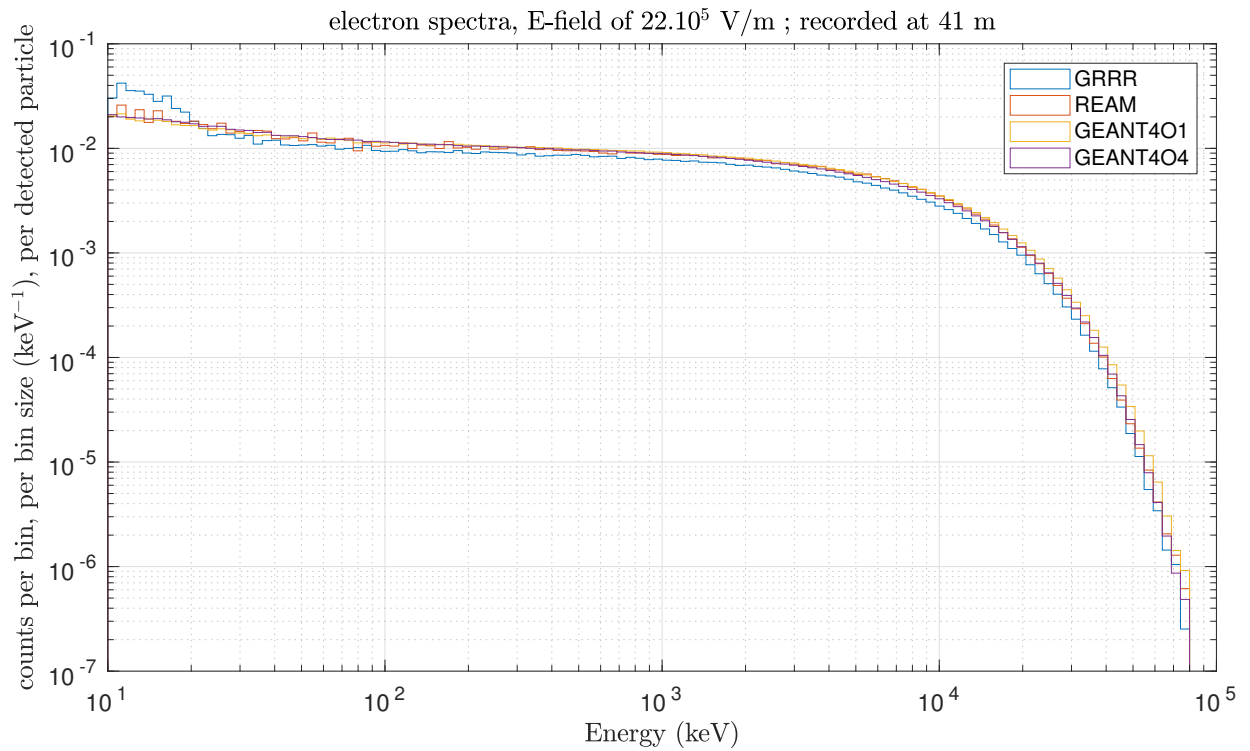
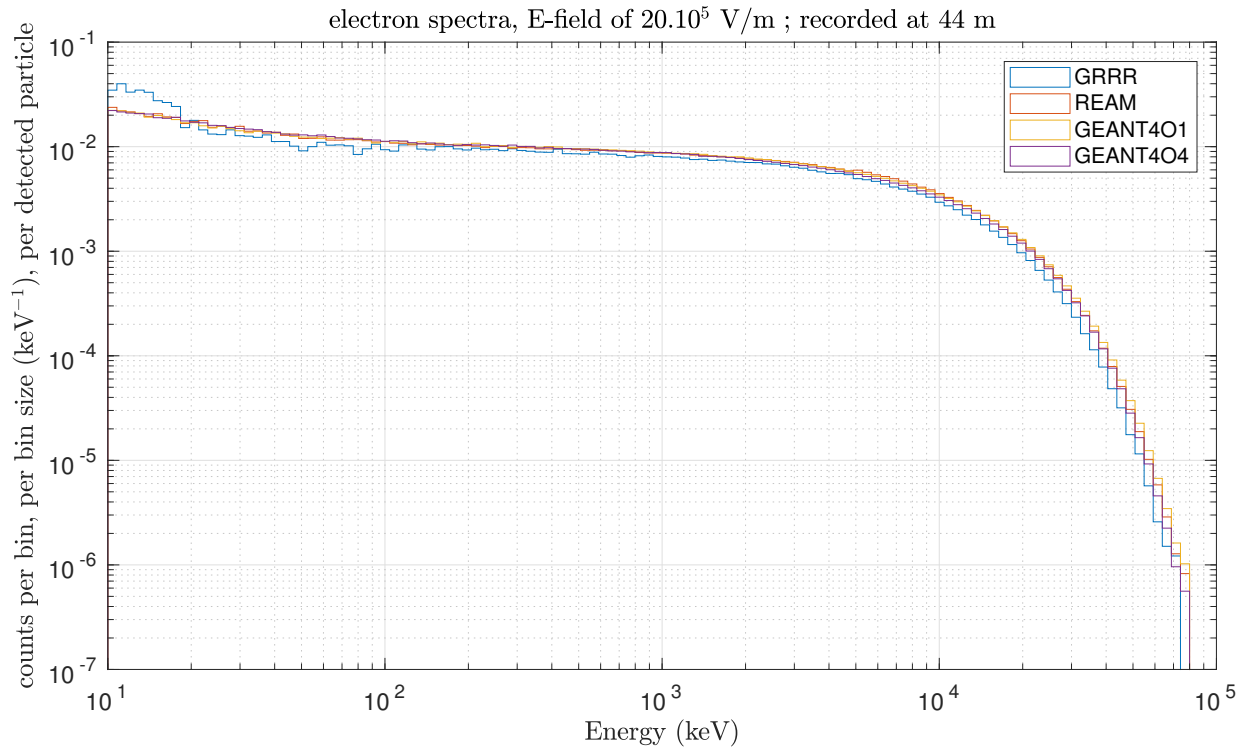


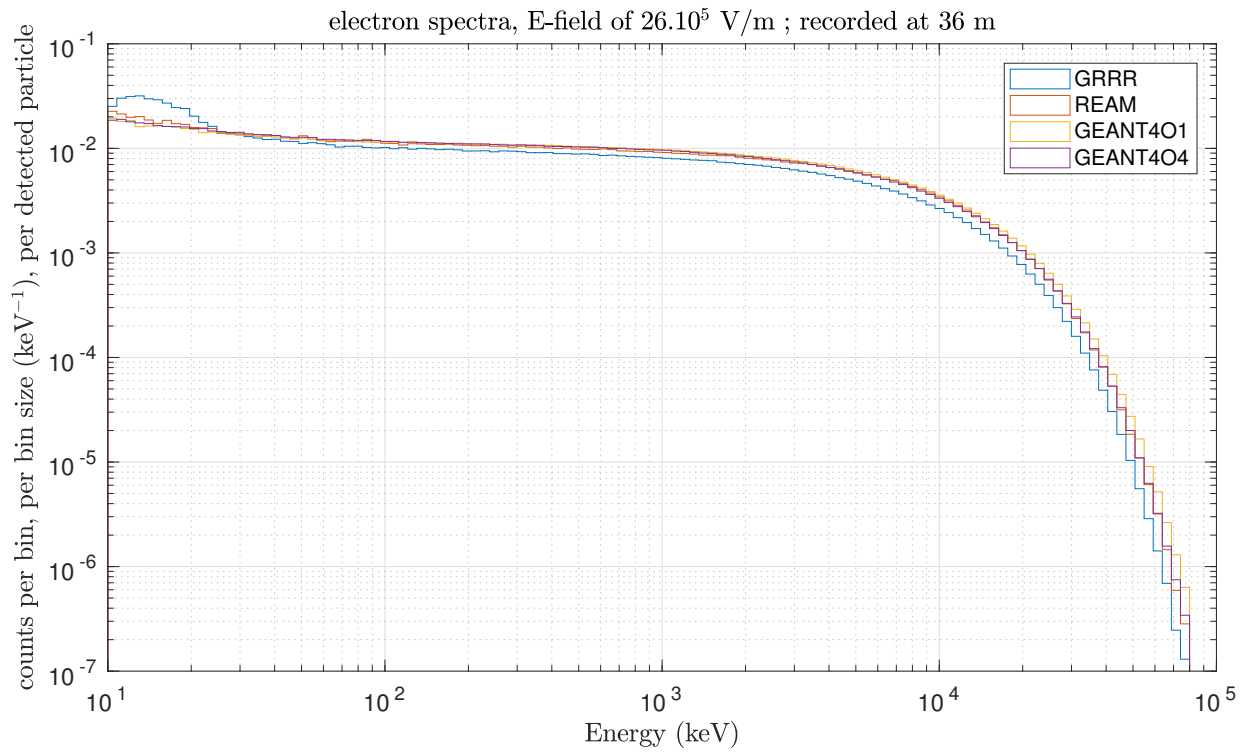
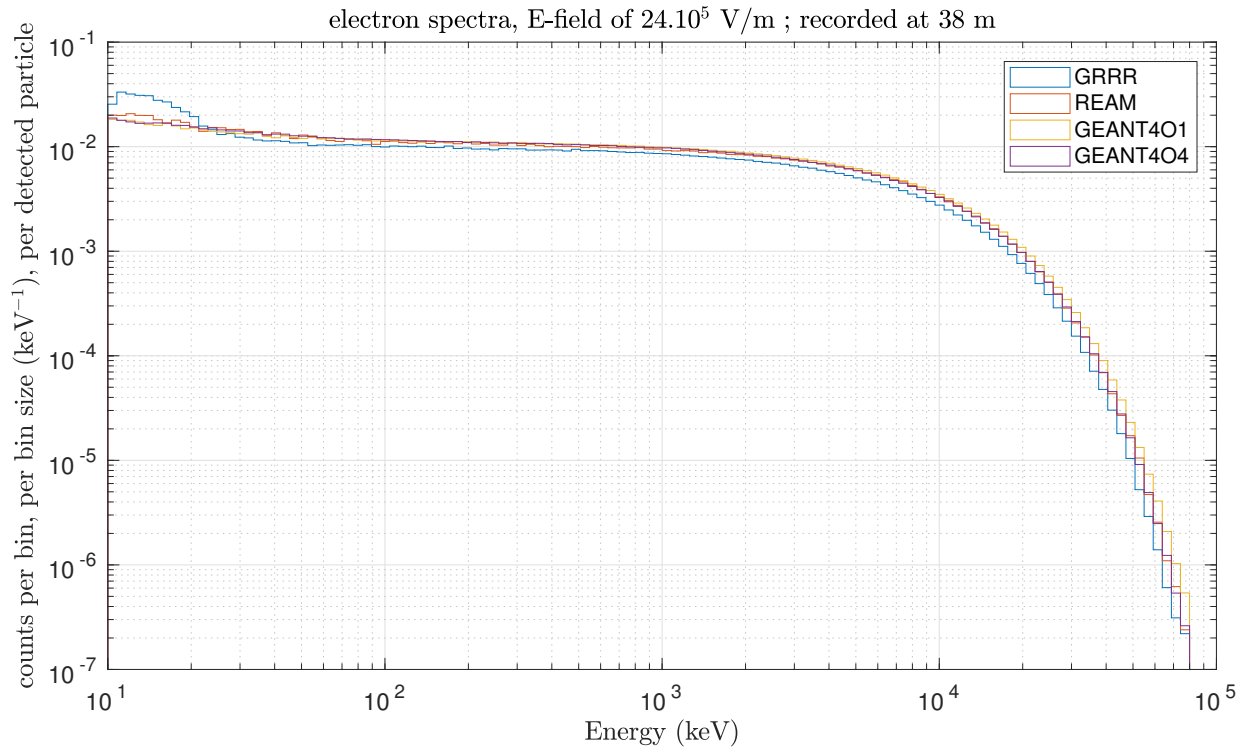


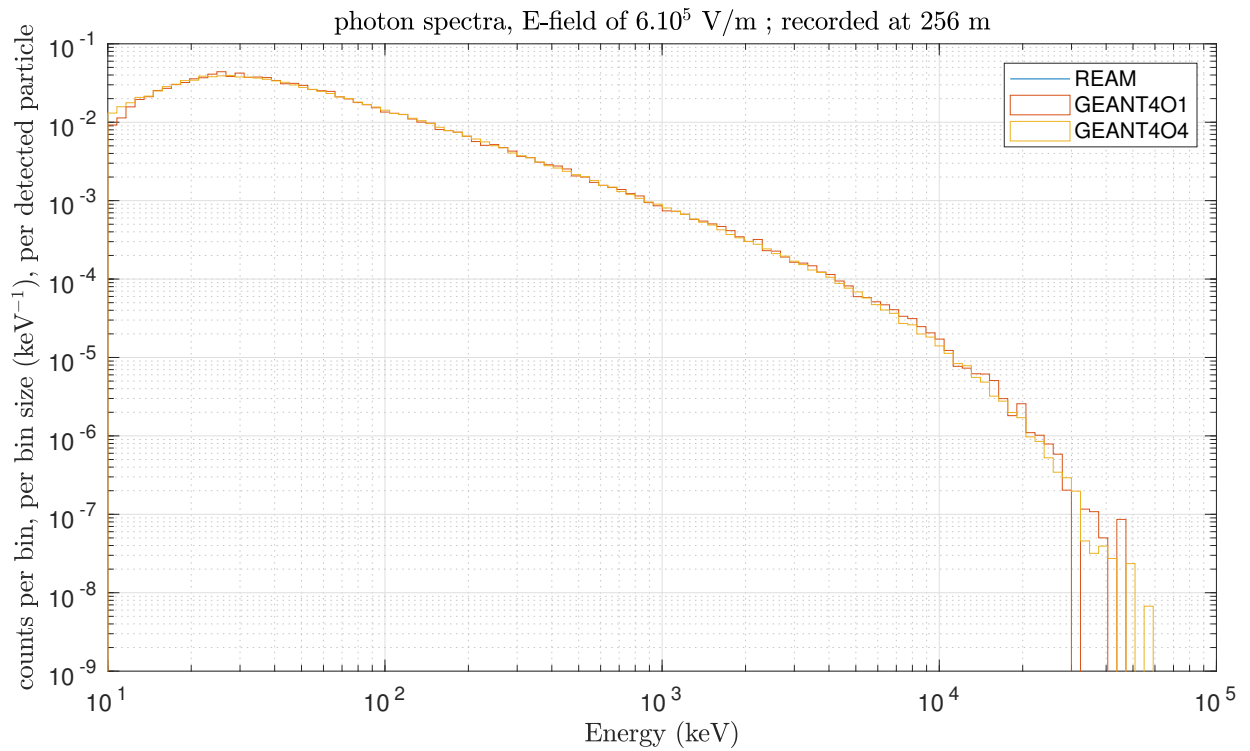
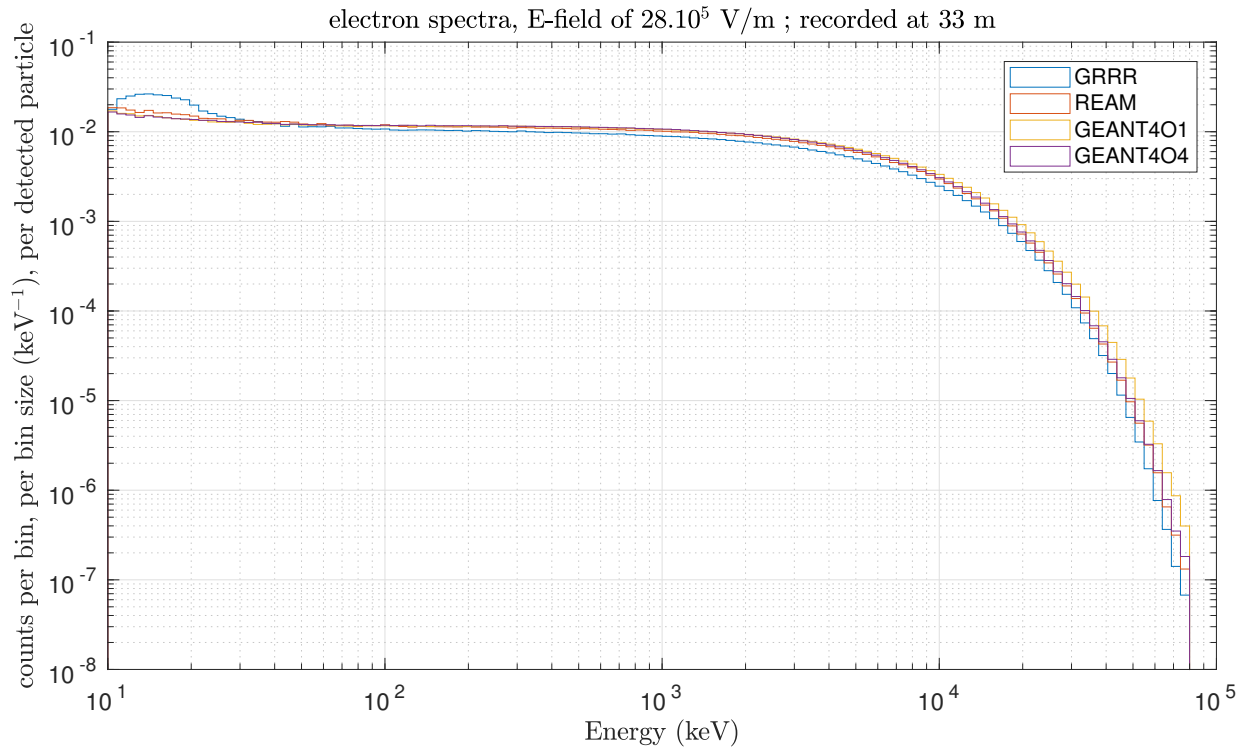


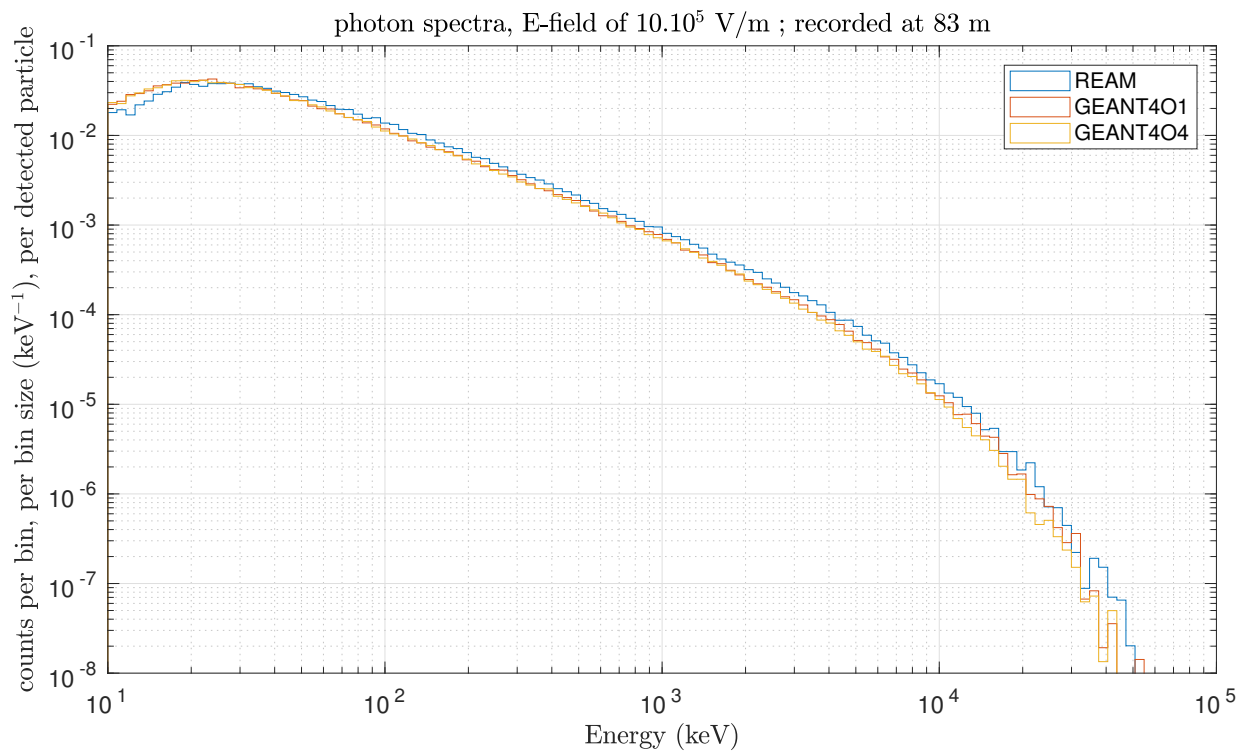
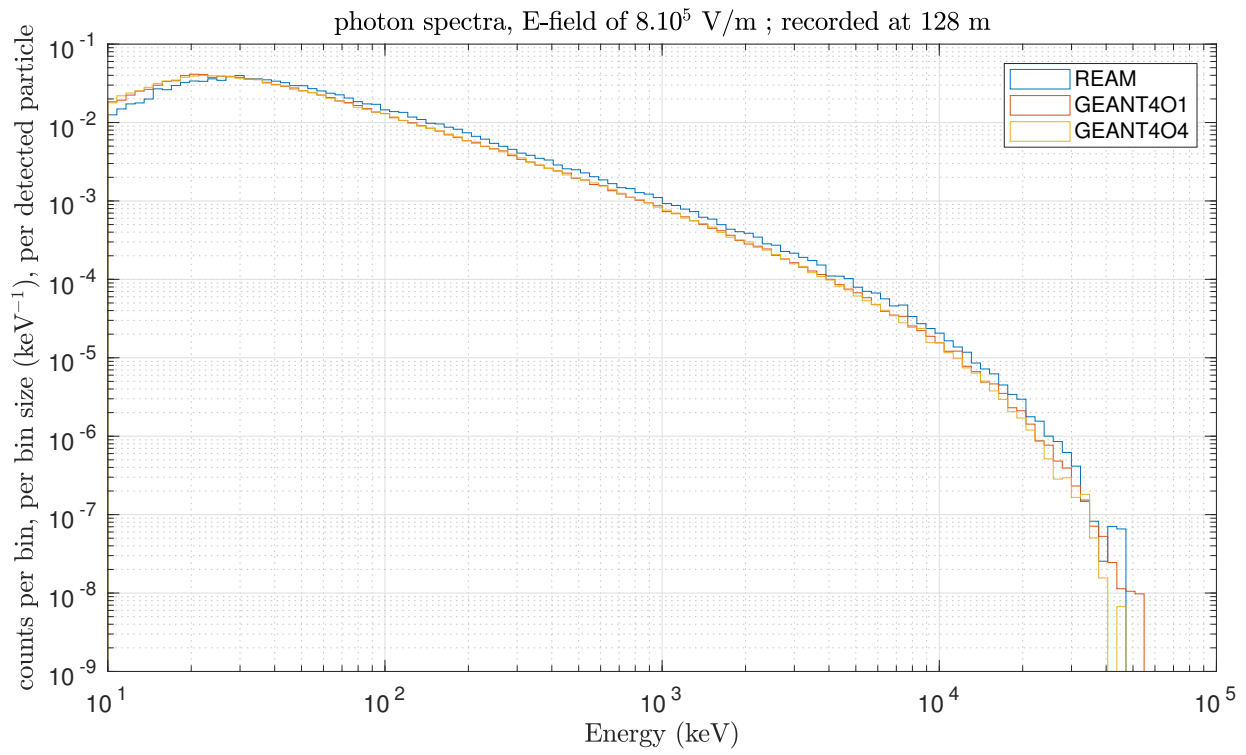


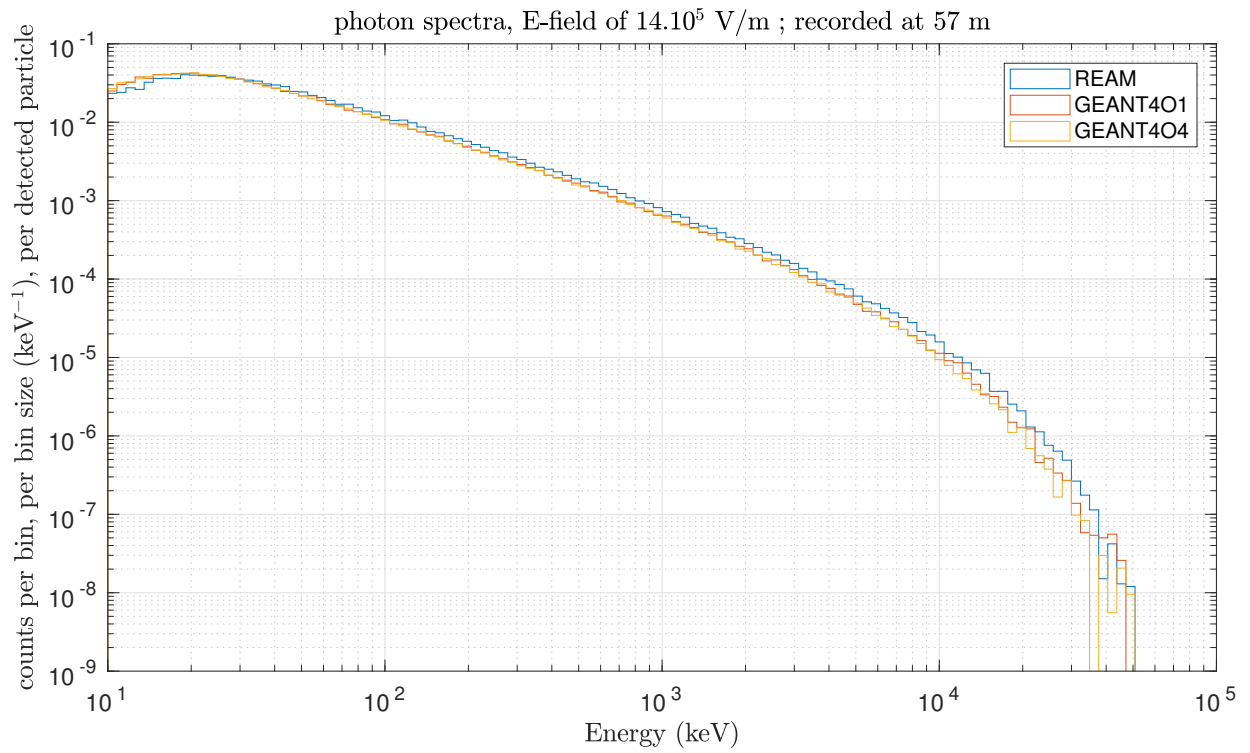
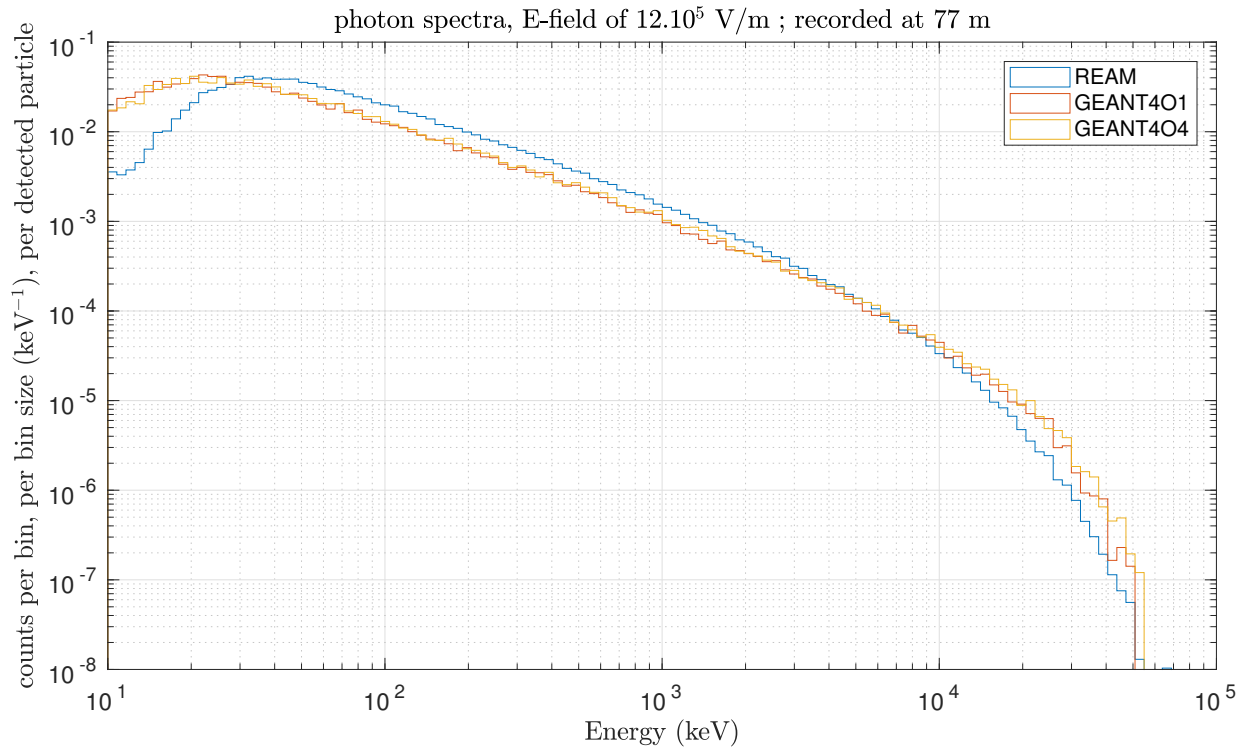




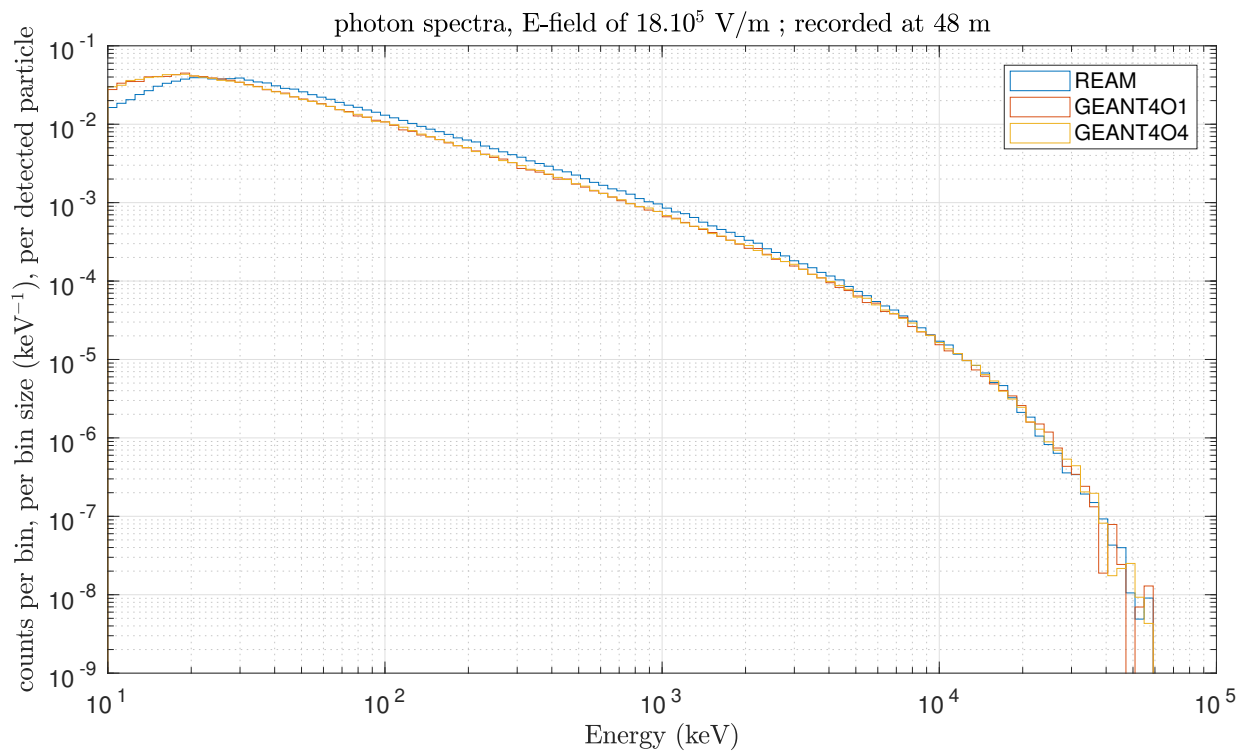
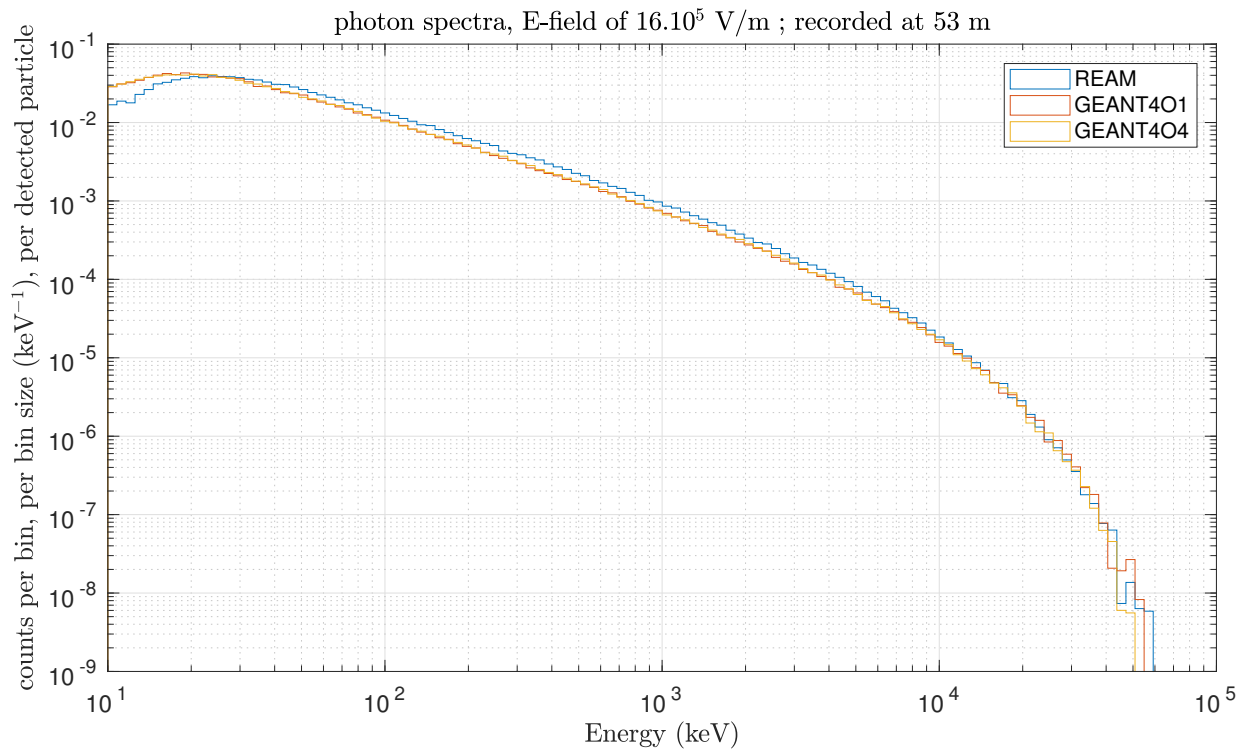


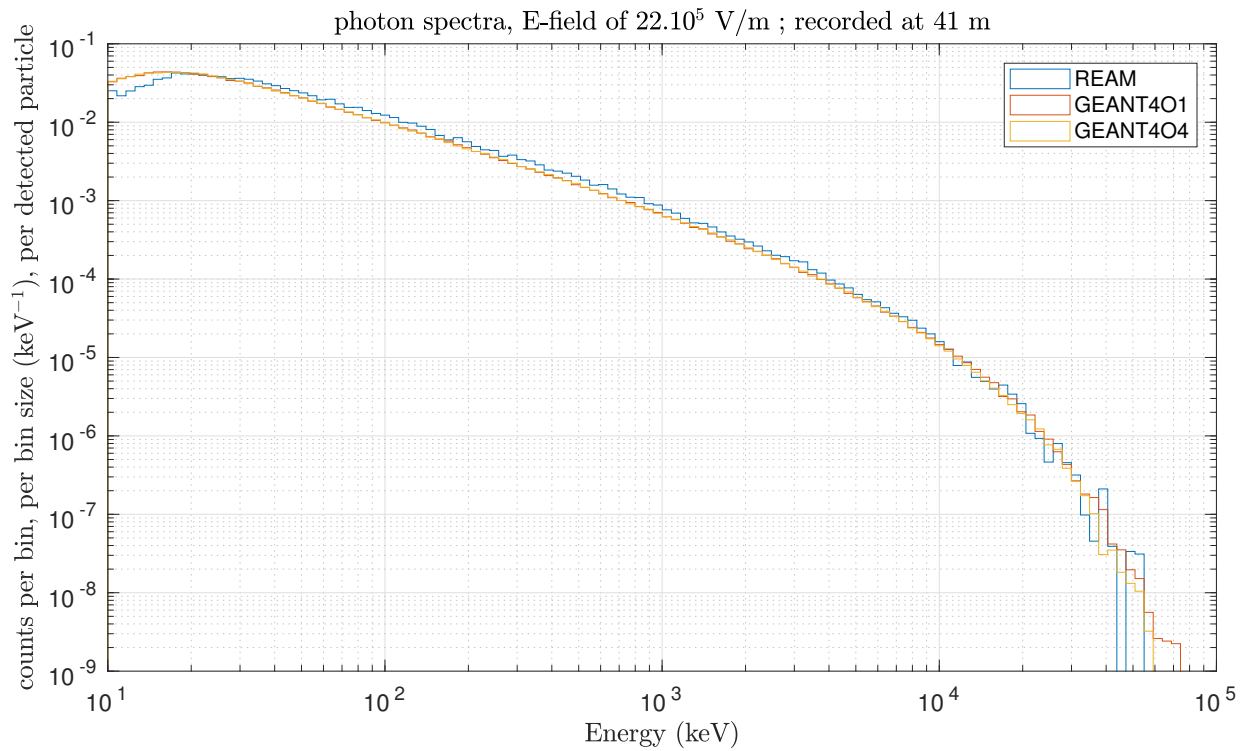
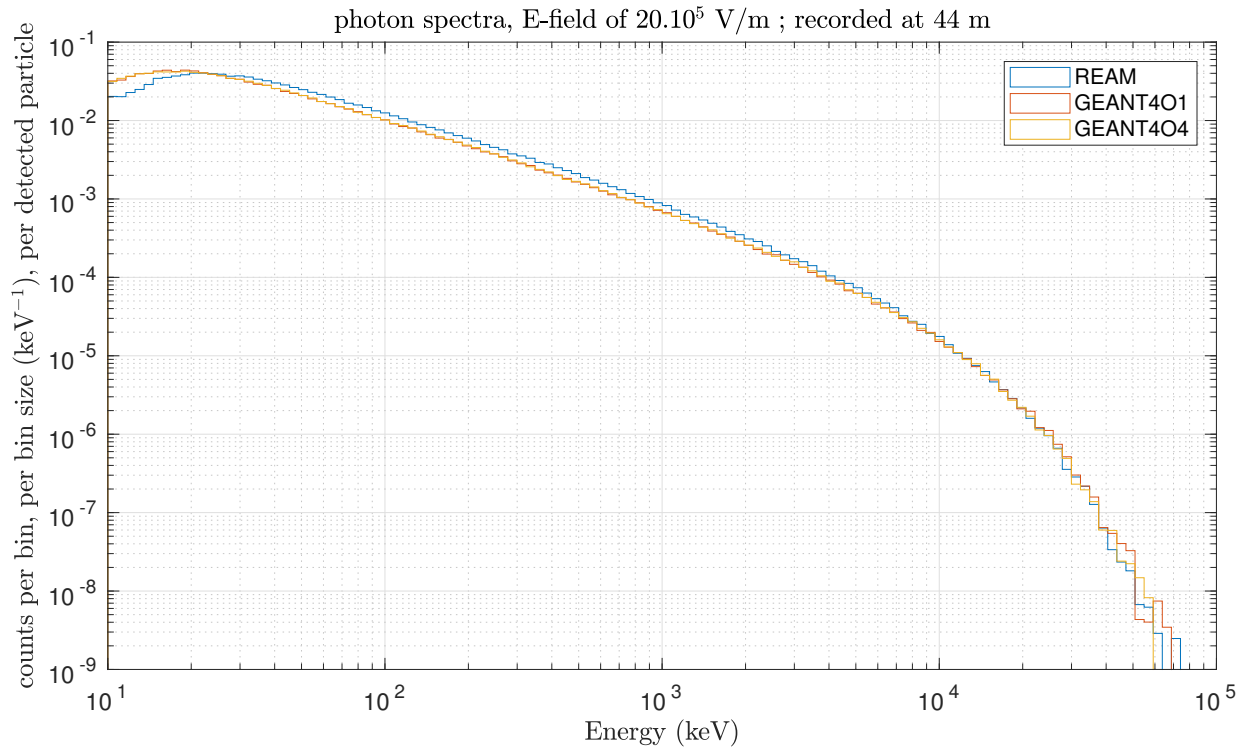


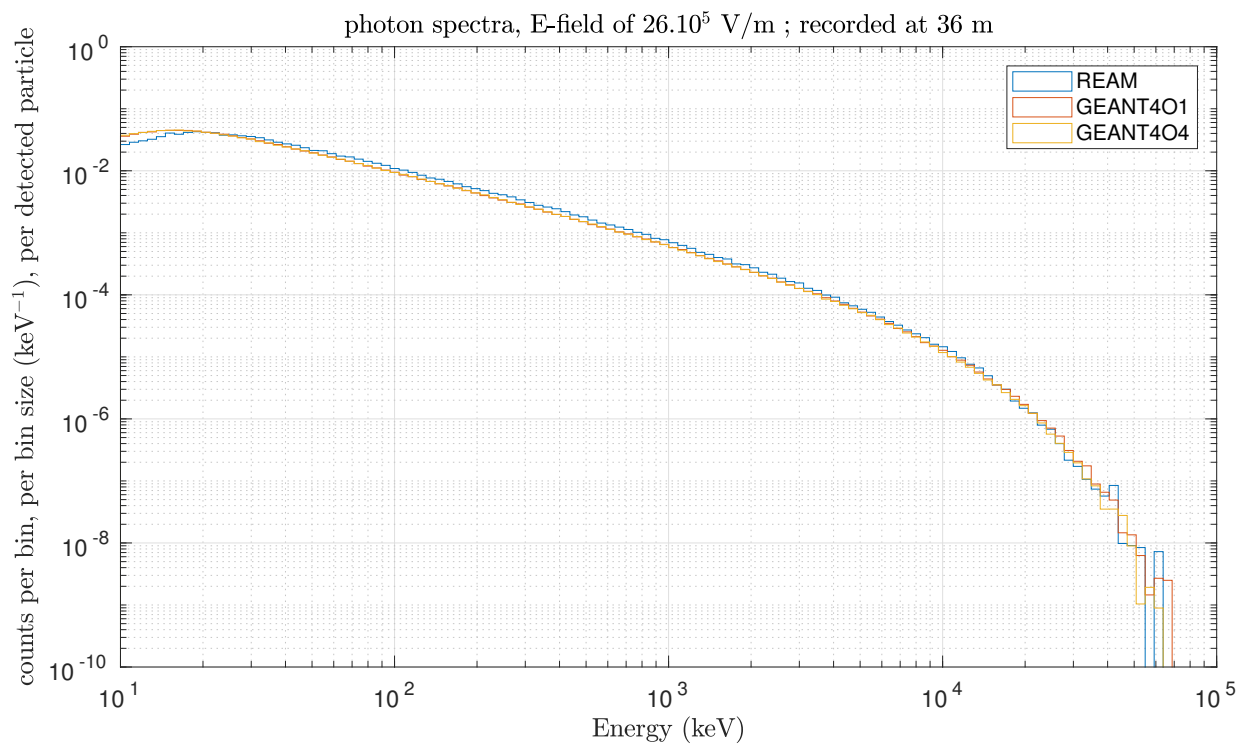
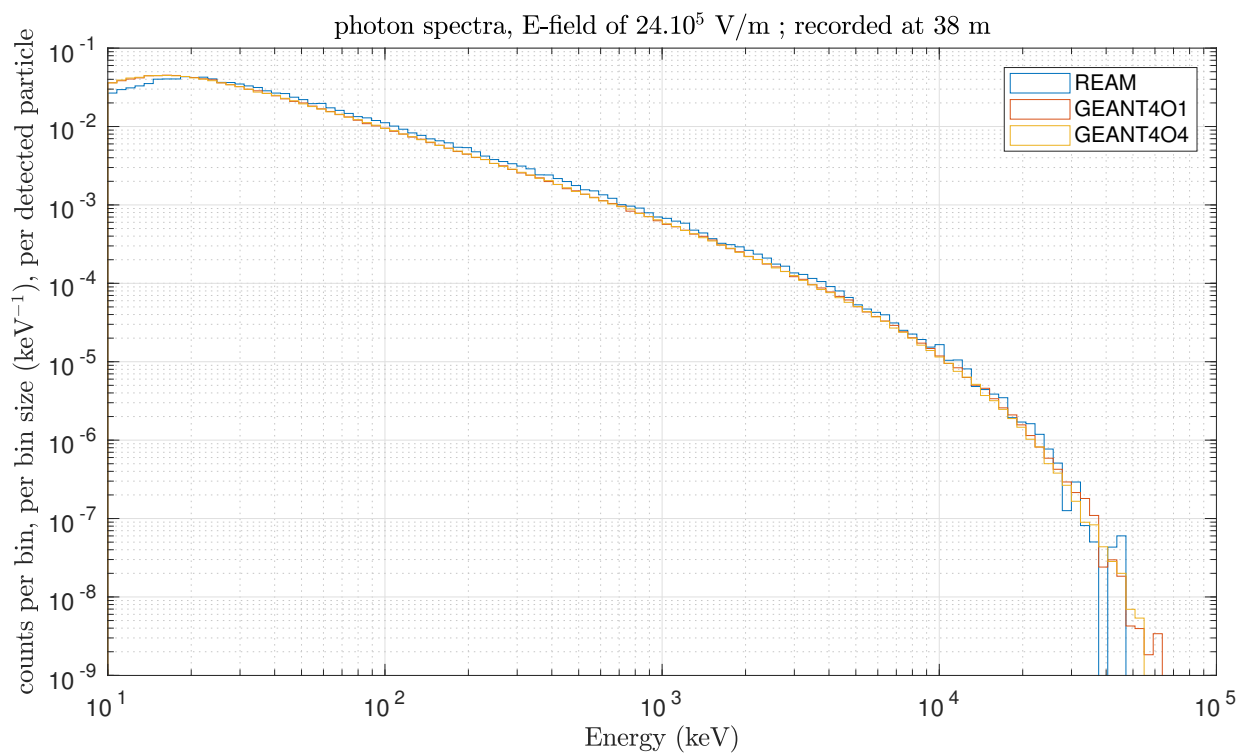


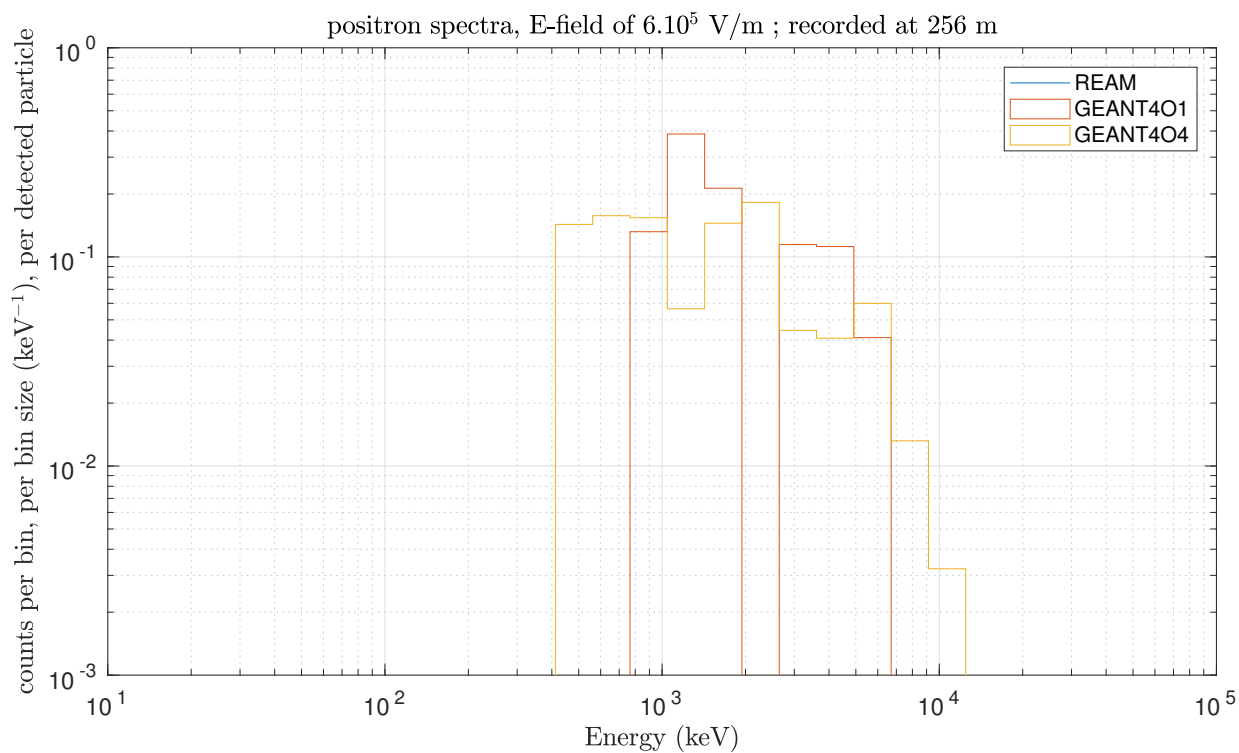
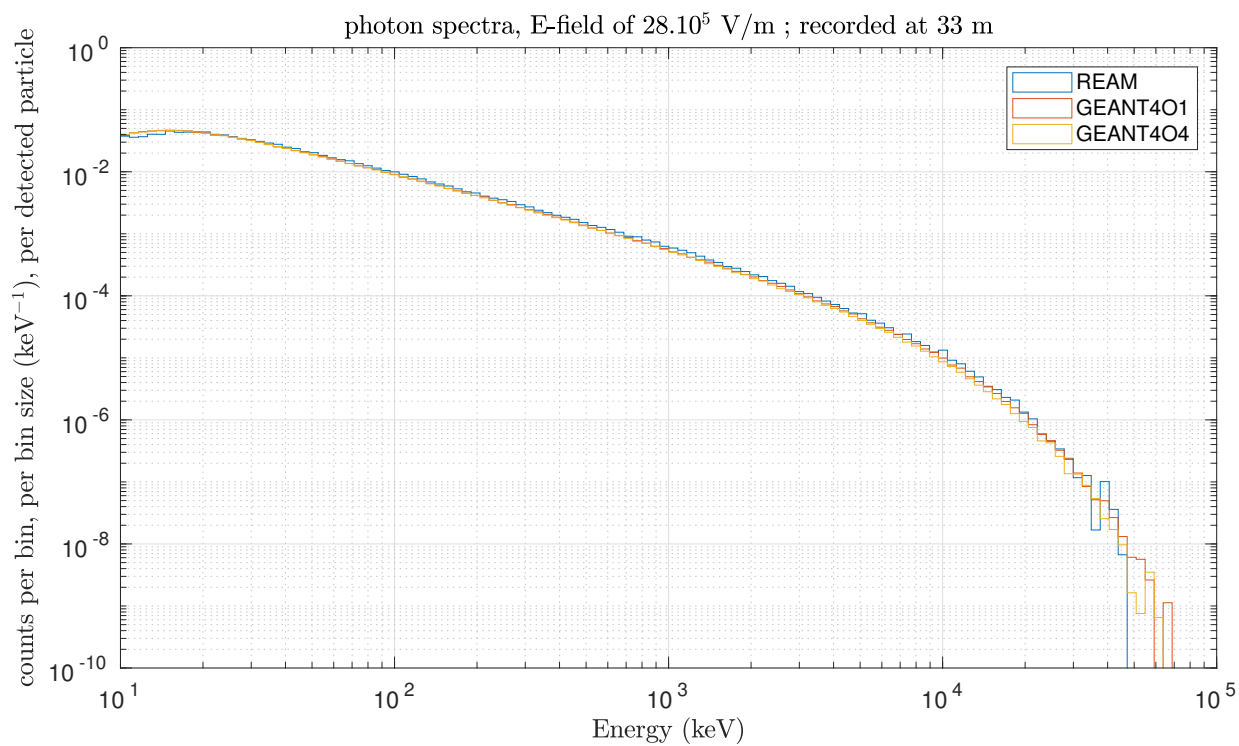


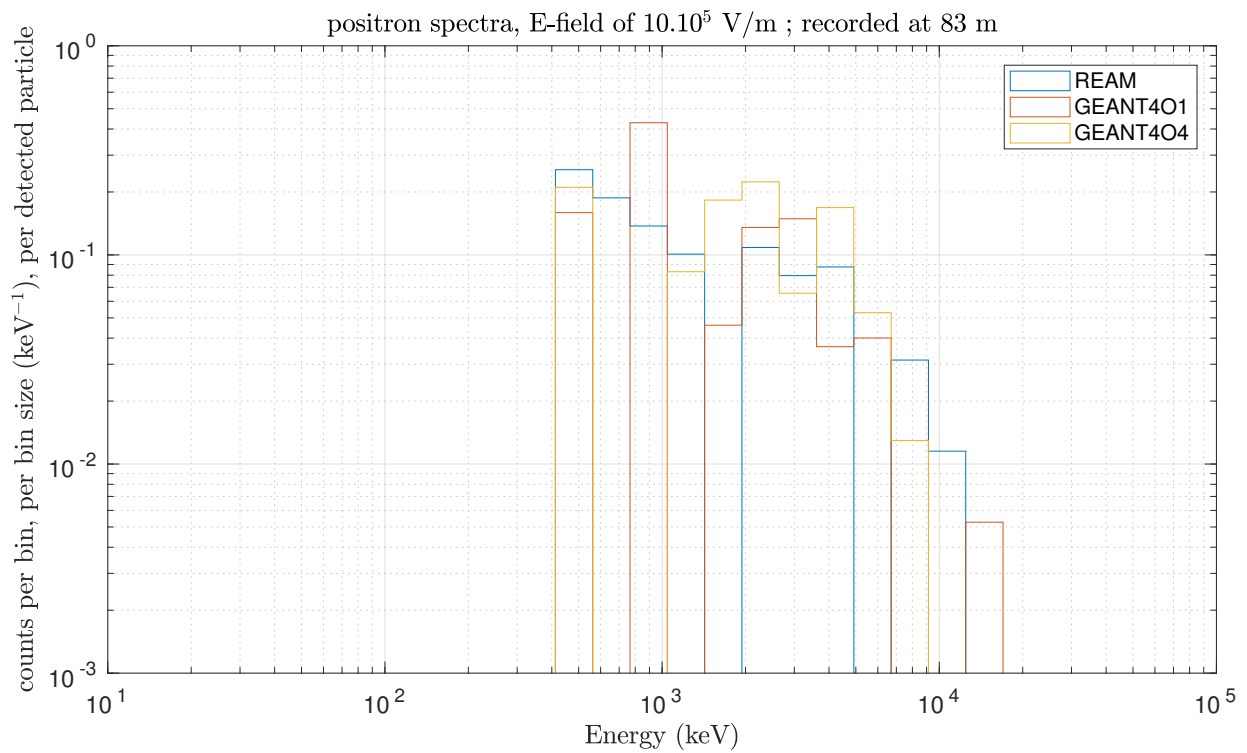
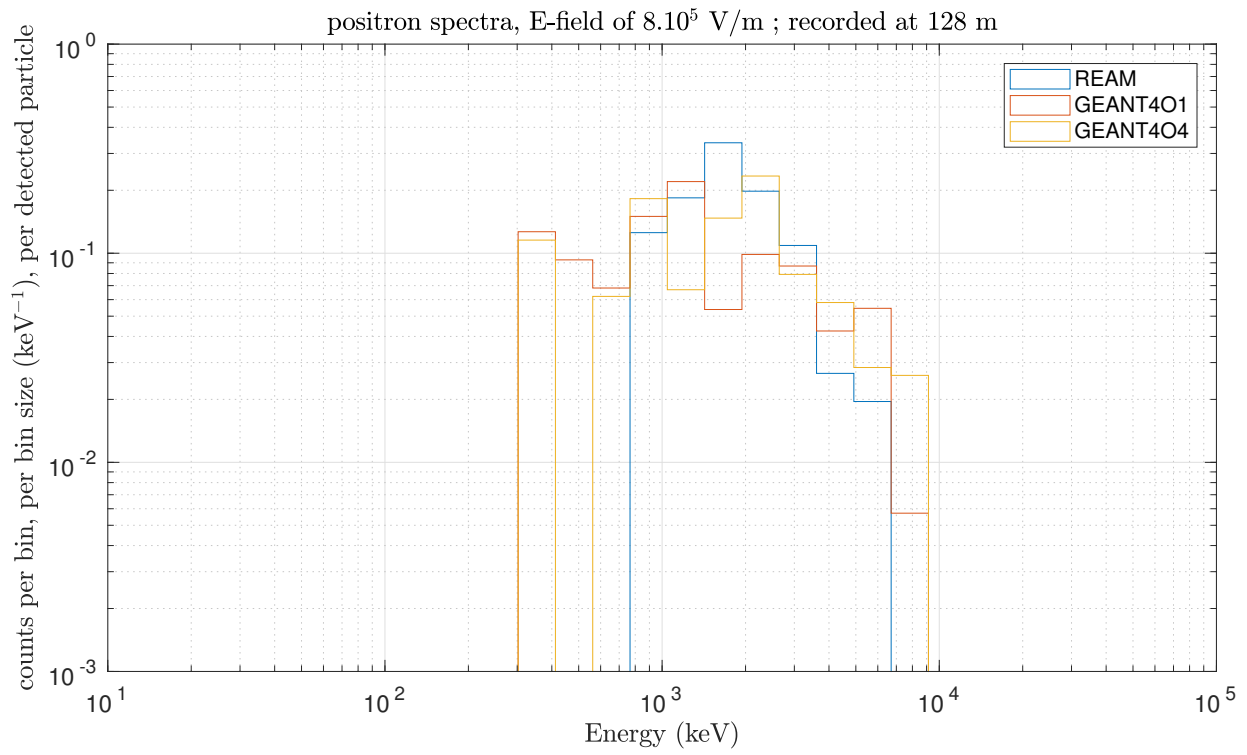


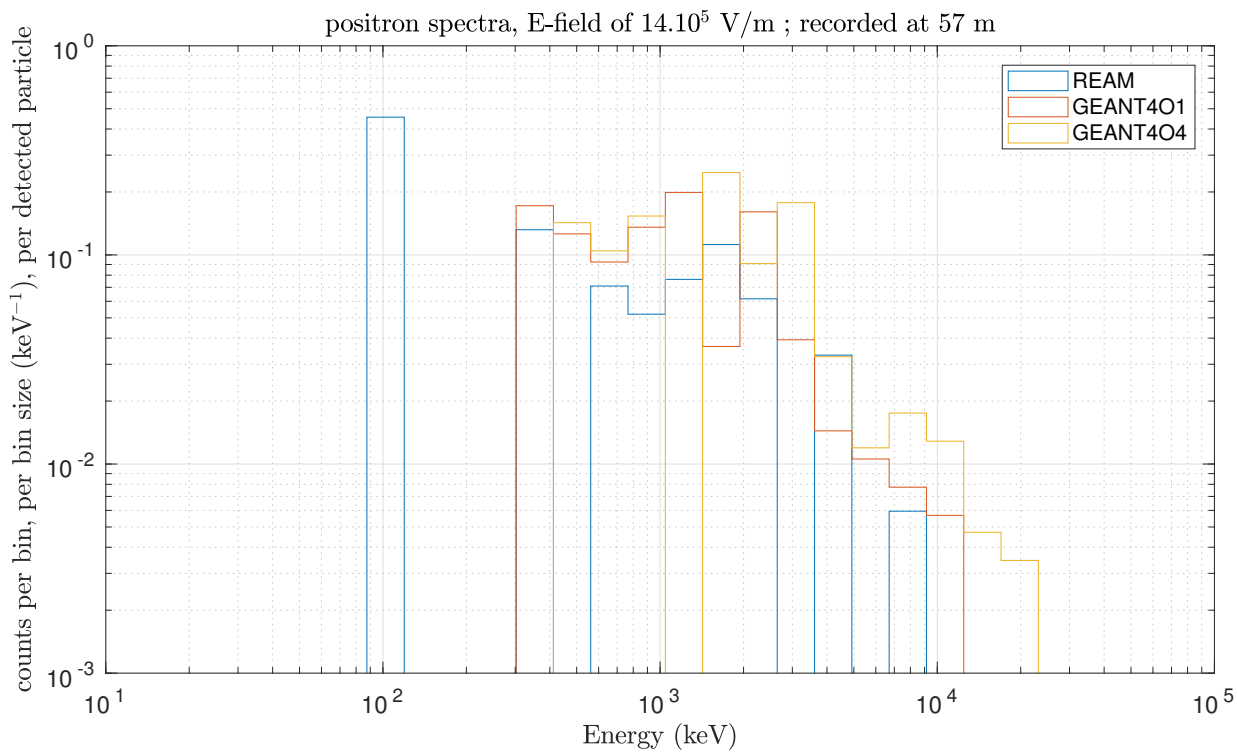
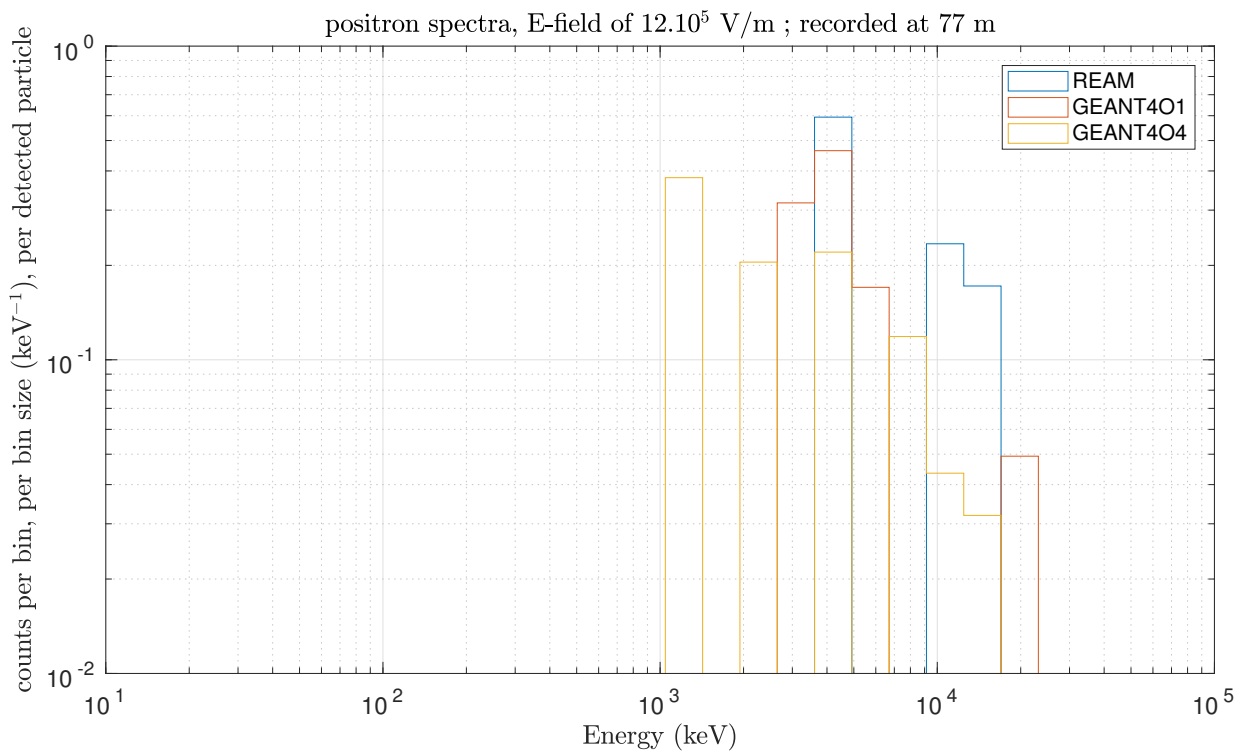


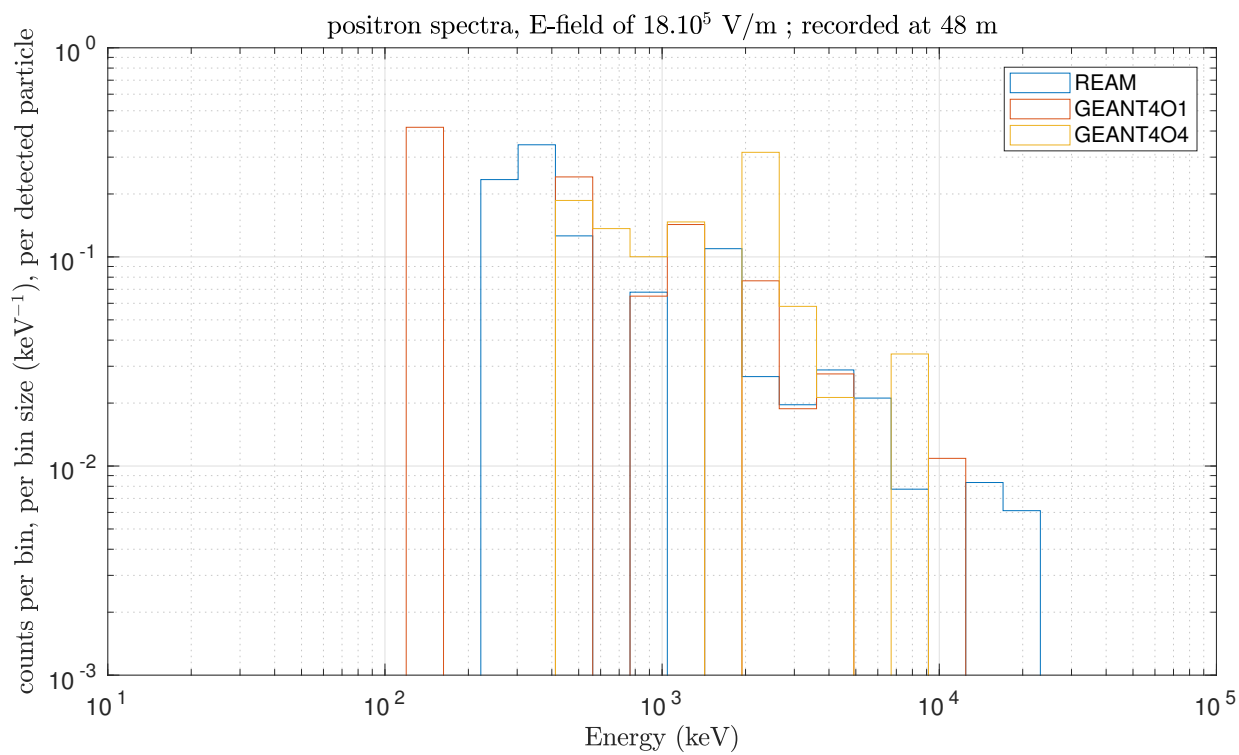
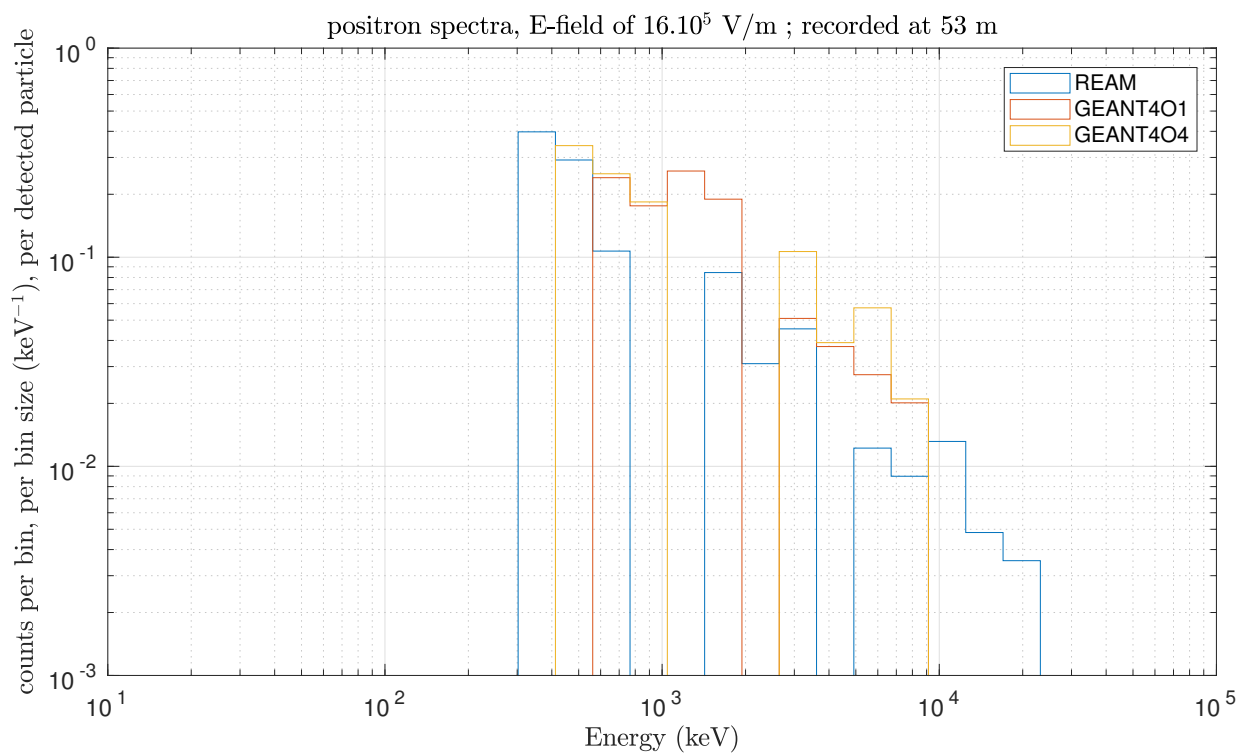


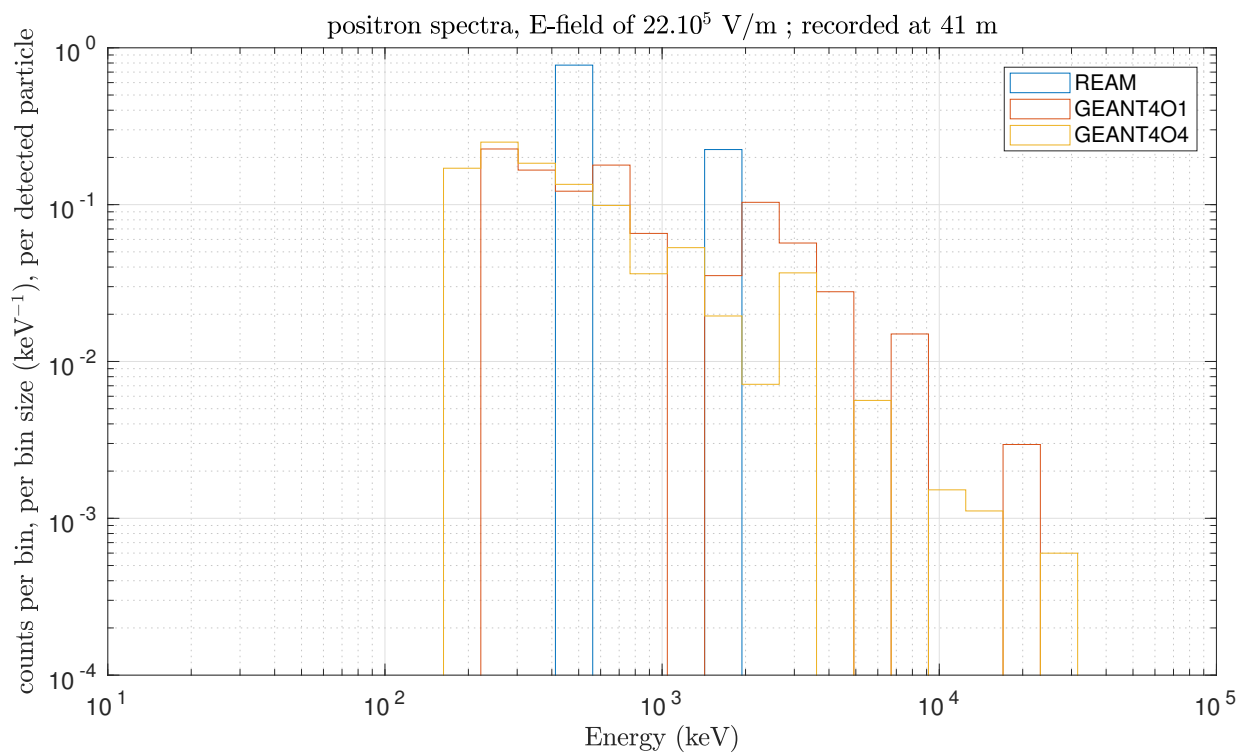
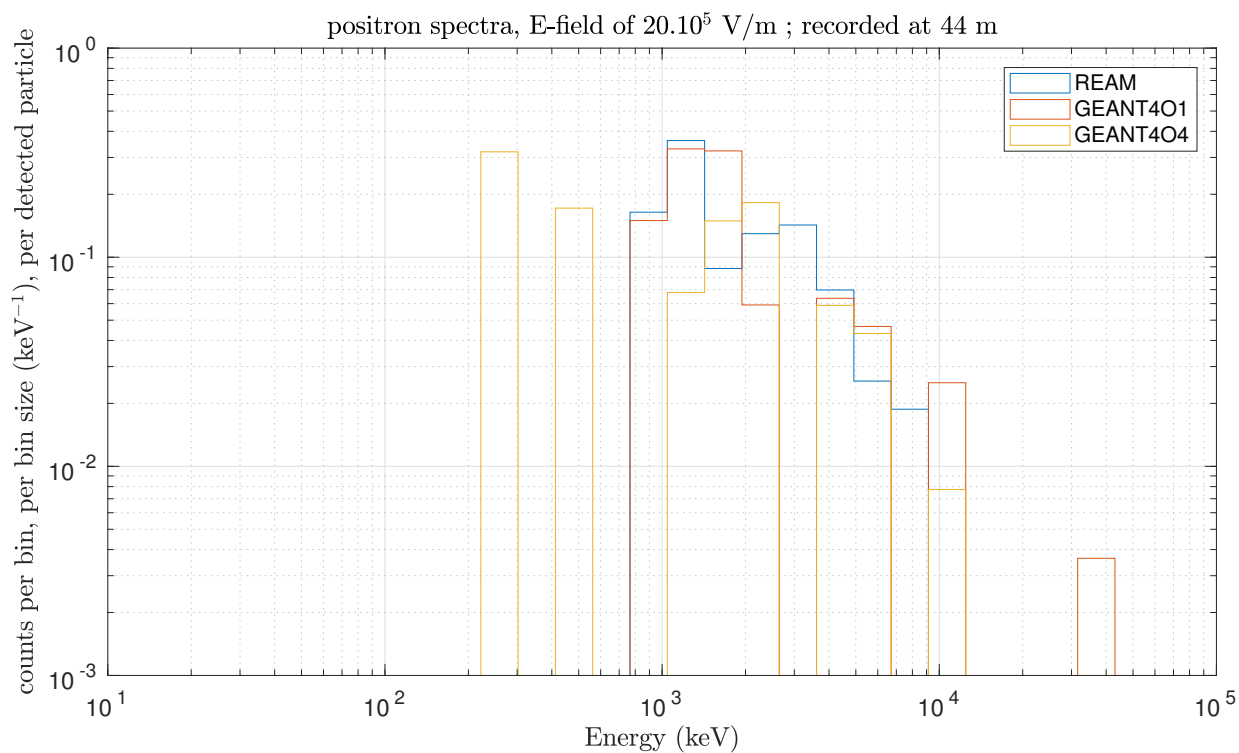




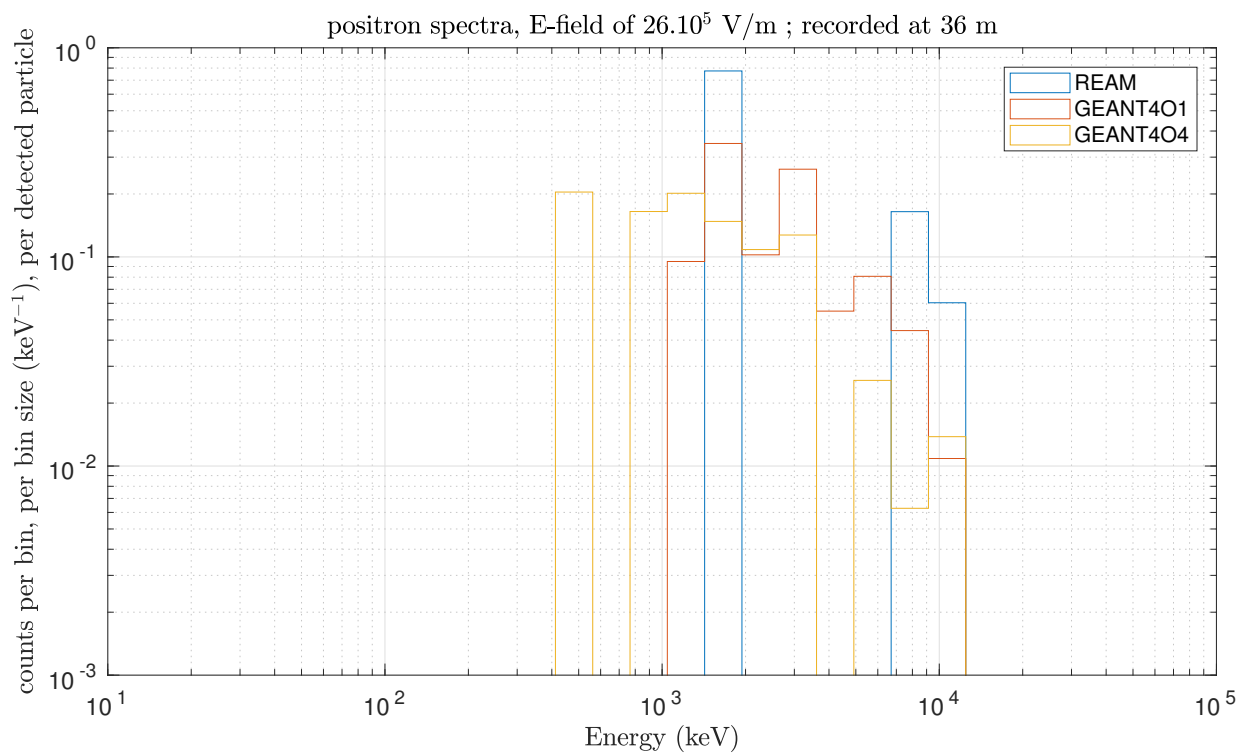
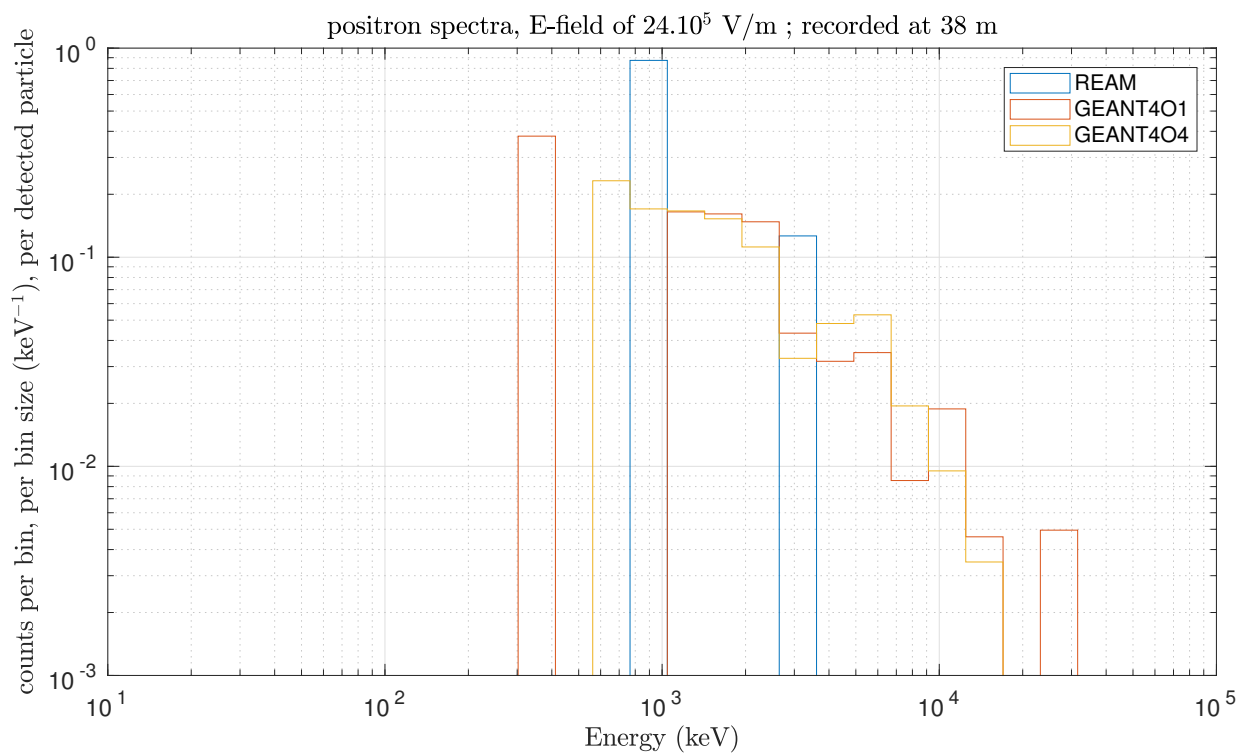


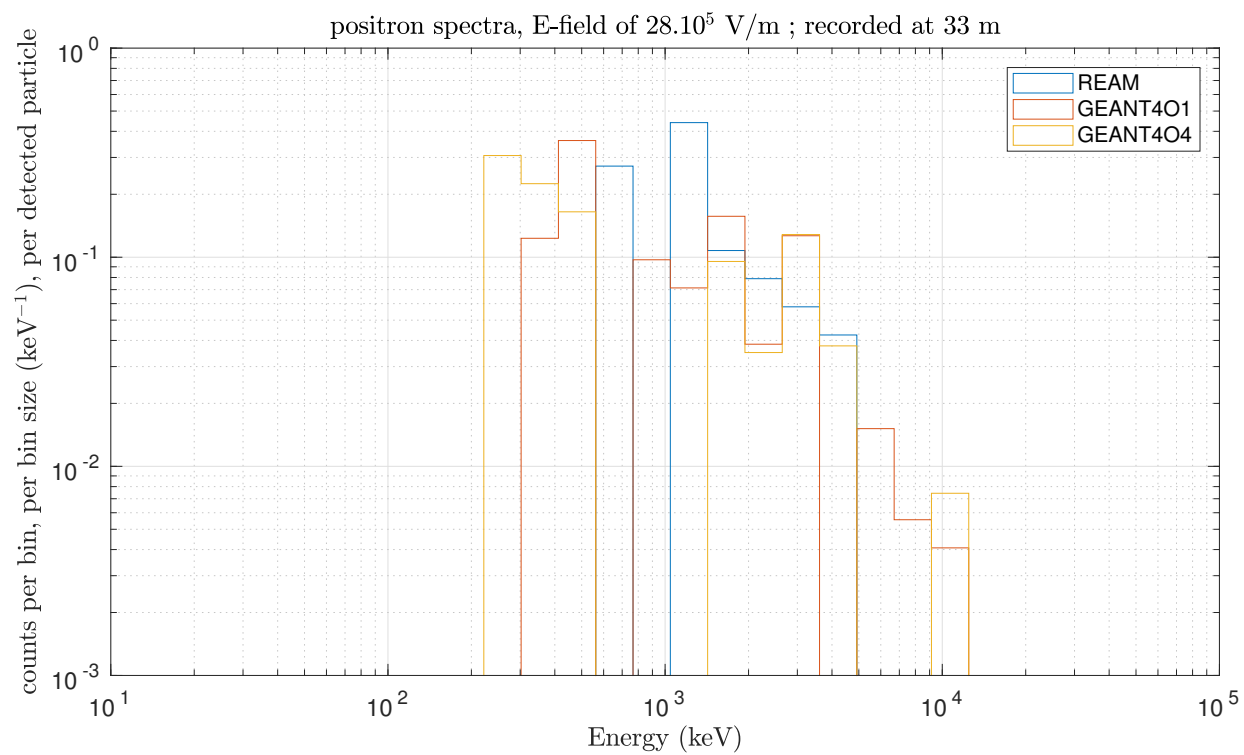




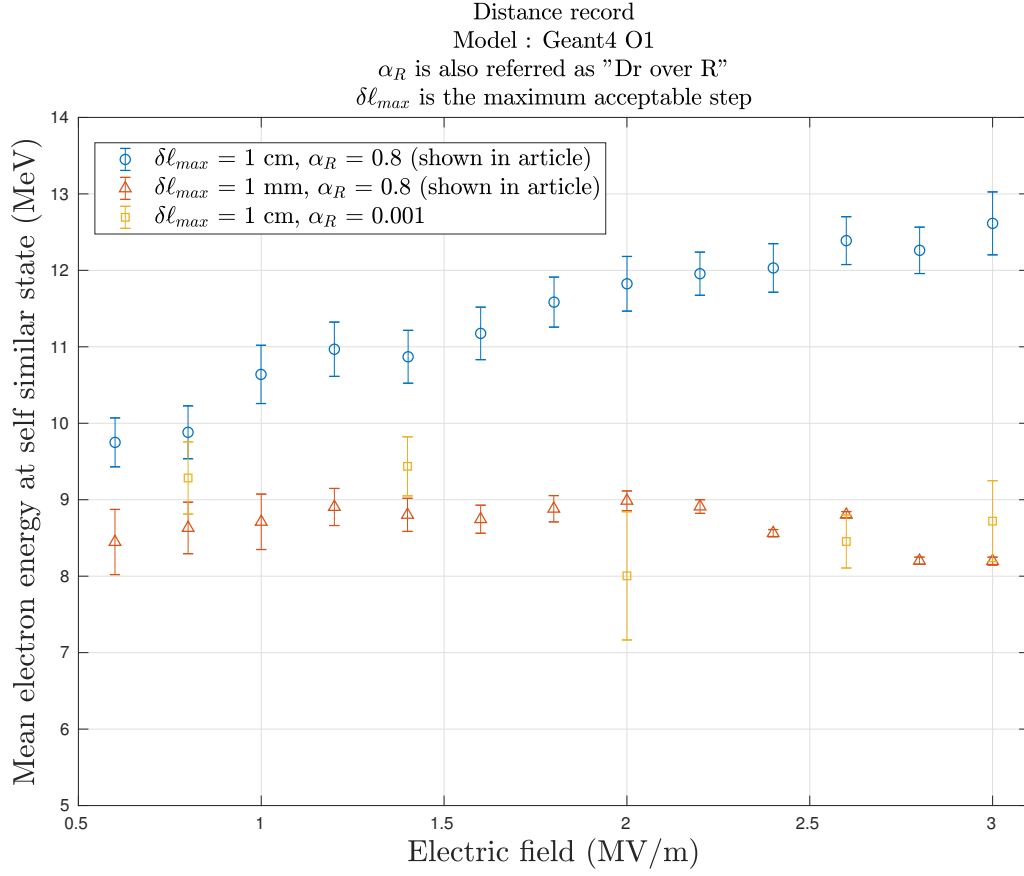






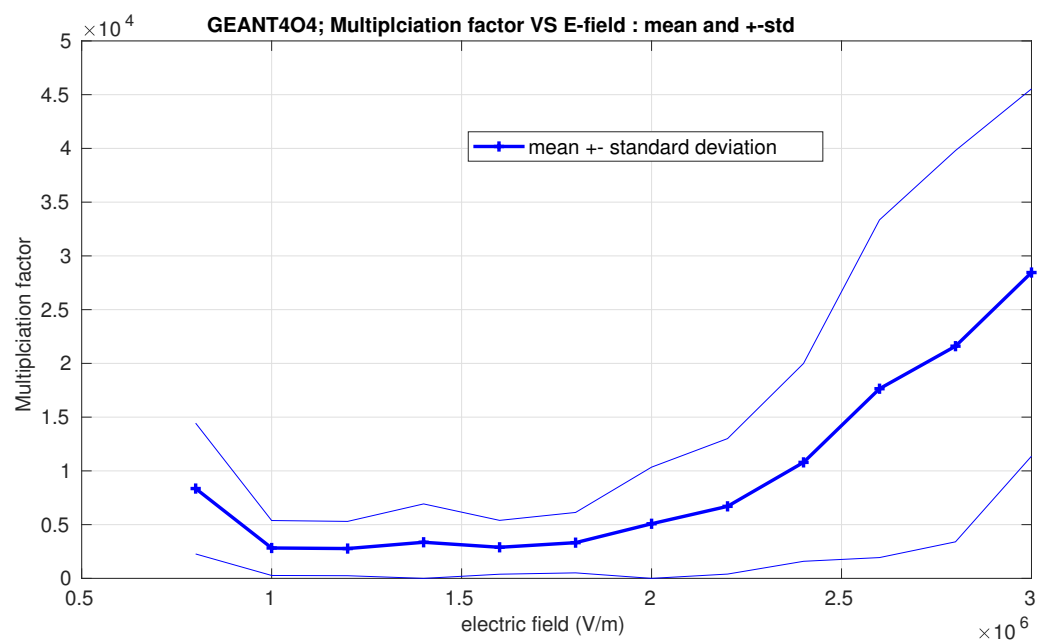


## 7 Electron Mean Energy VS E-field, with $\alpha_R$ ("Dr Over R")

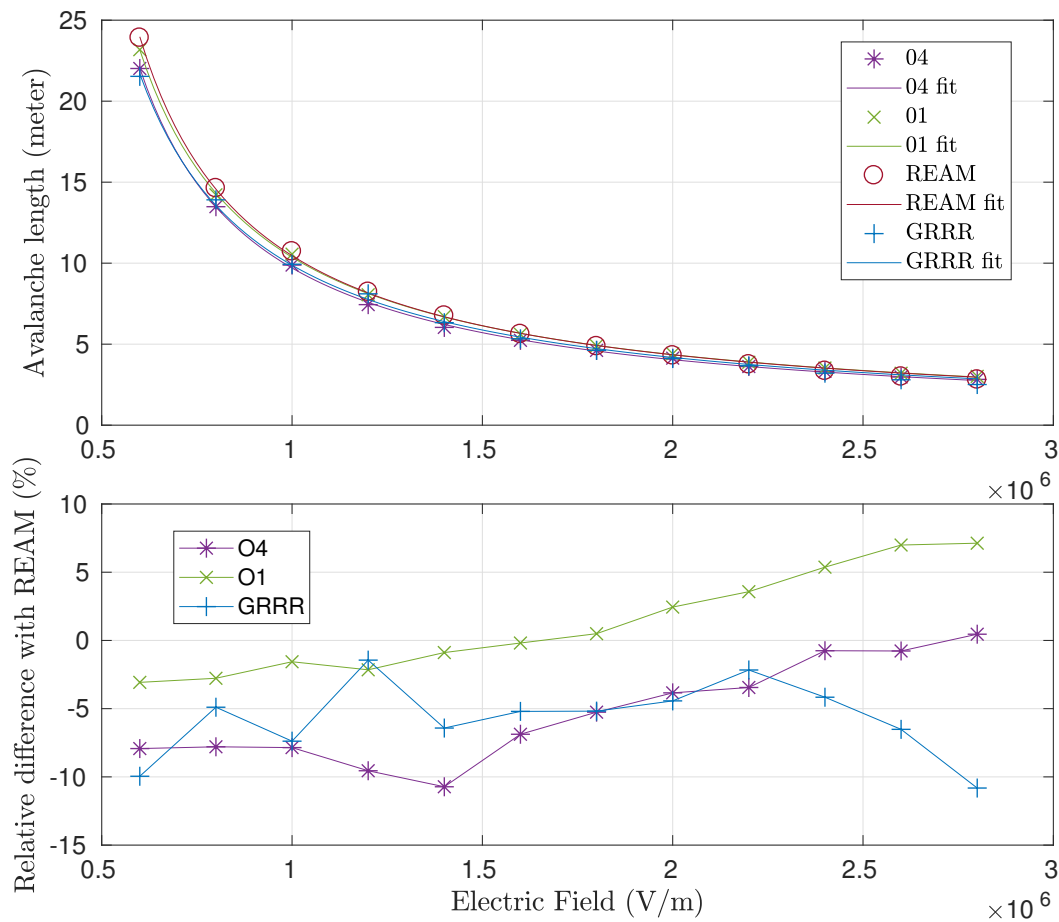


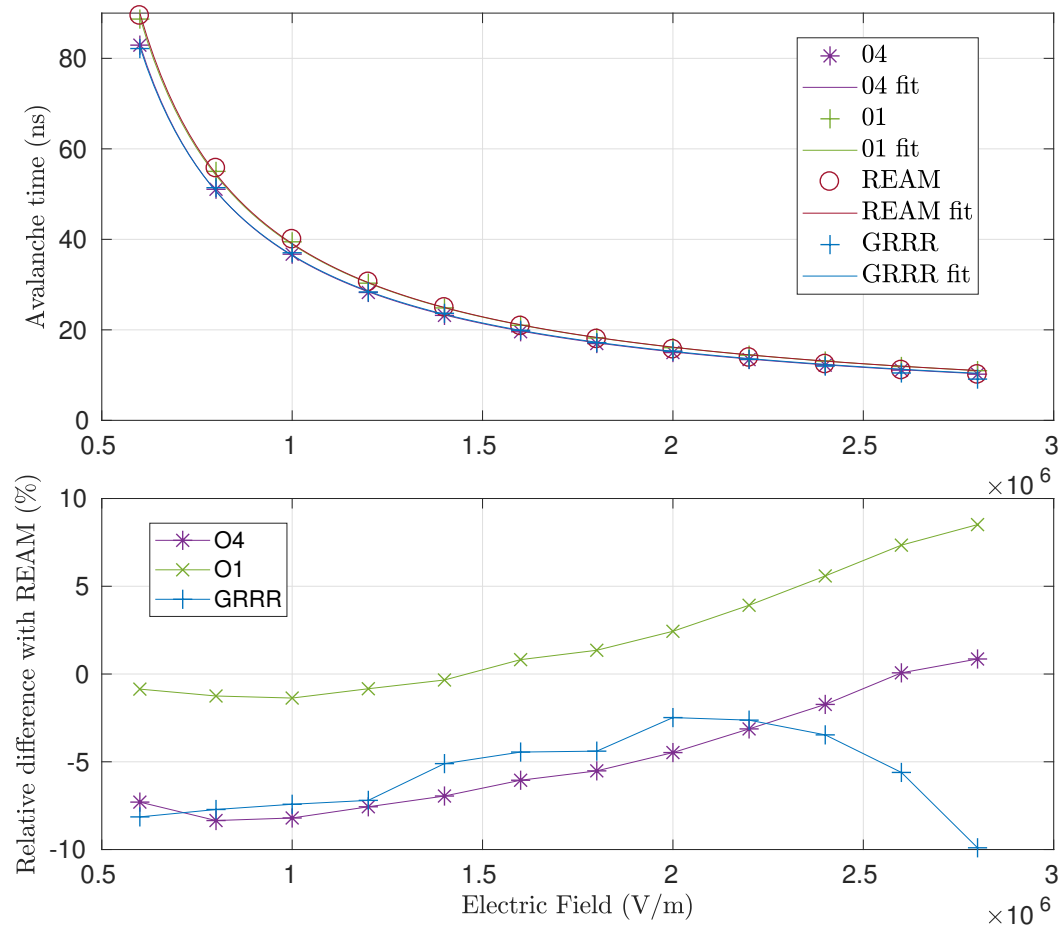
8 Multiplication Factors

8.1 Shower To Shower Fluctuations

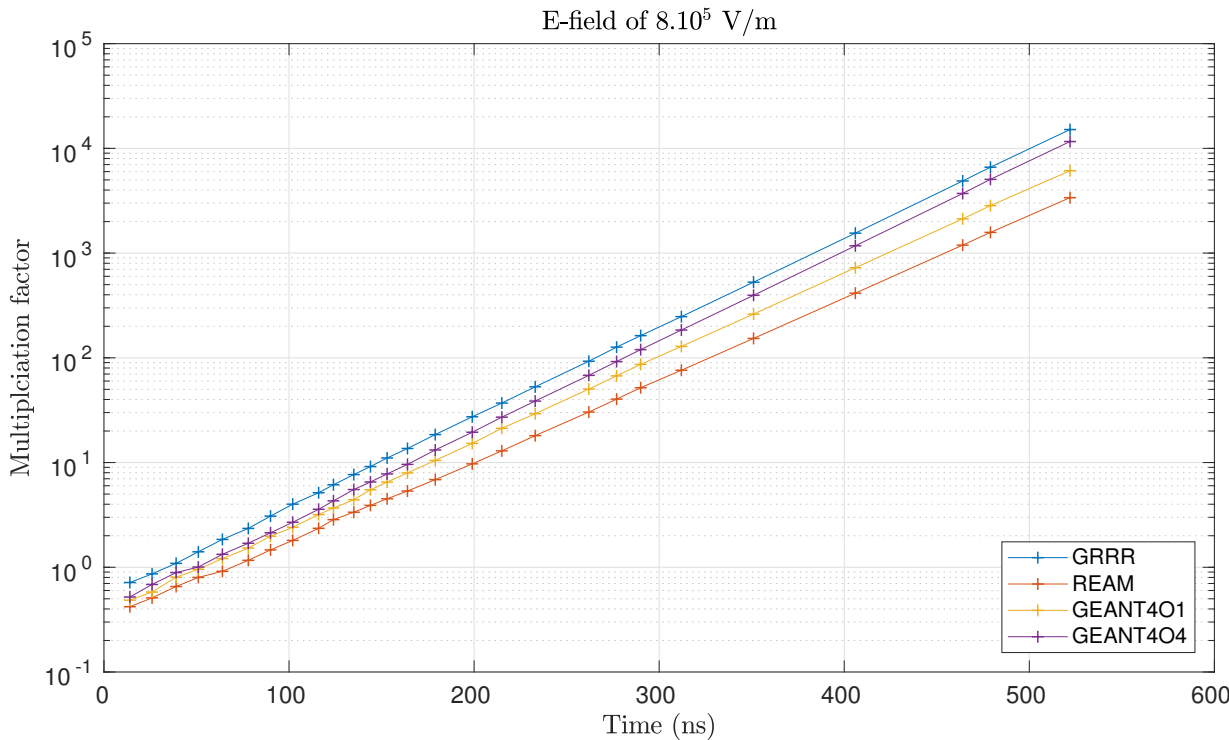
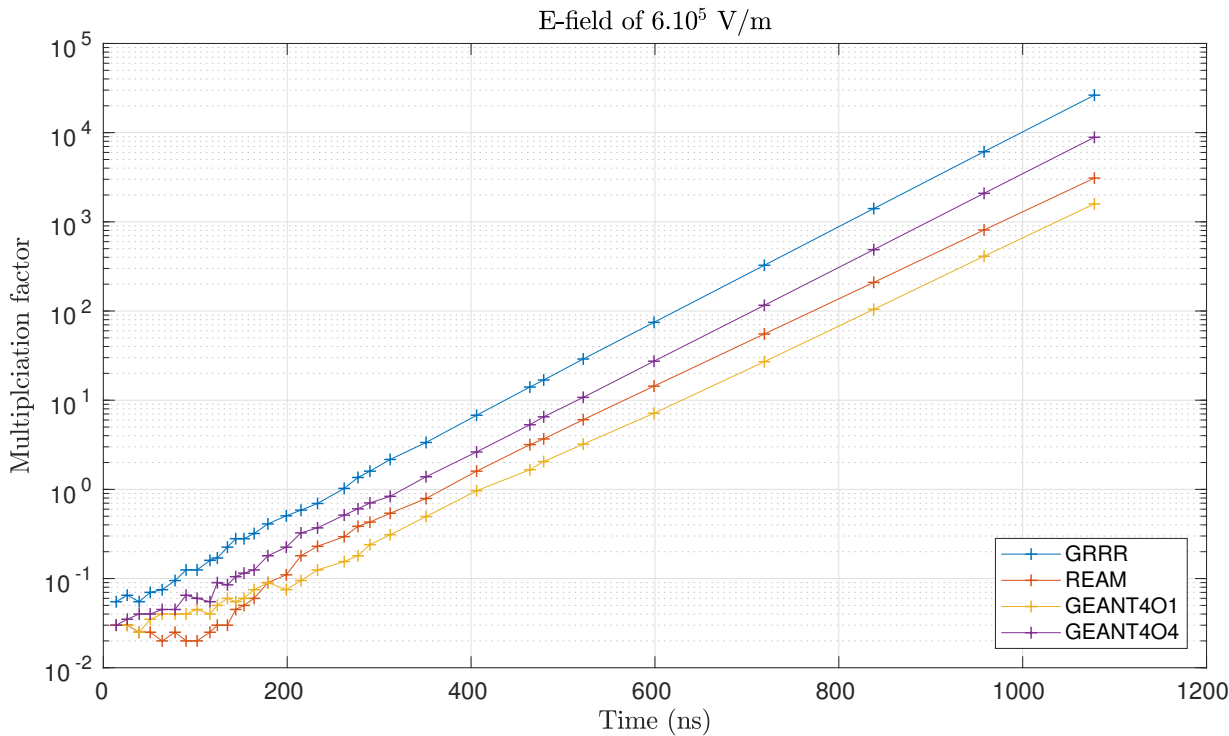


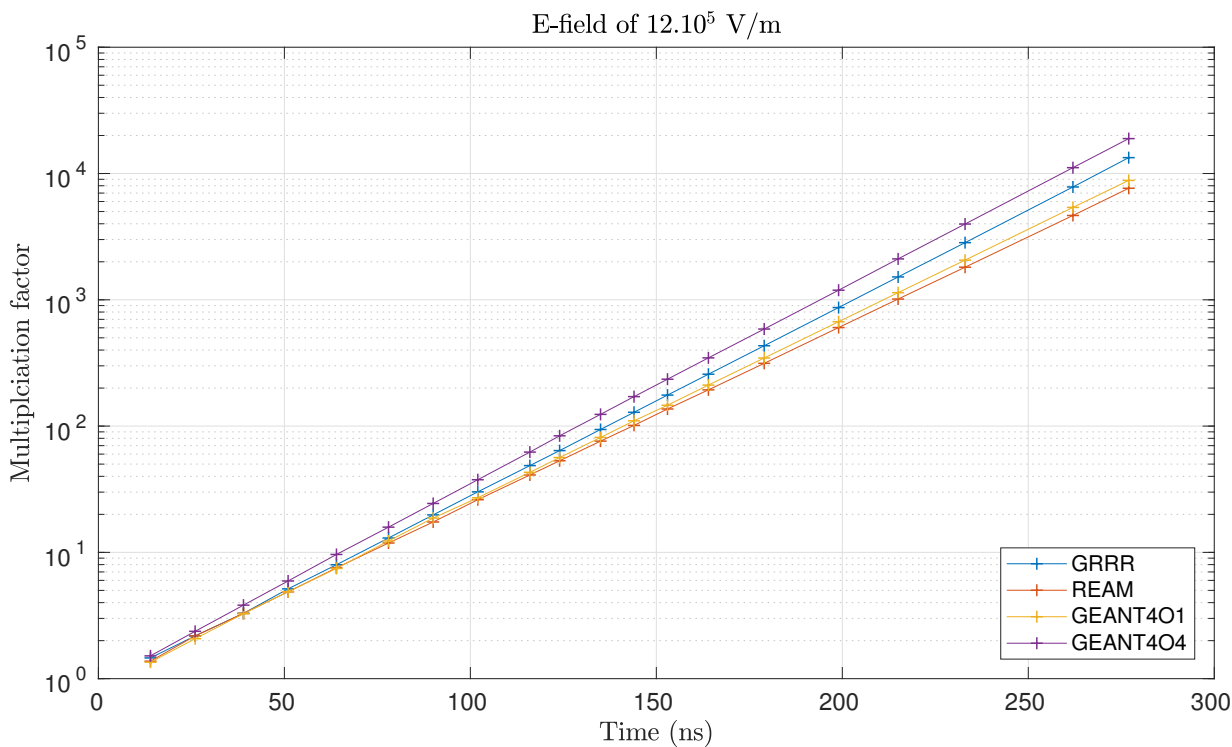
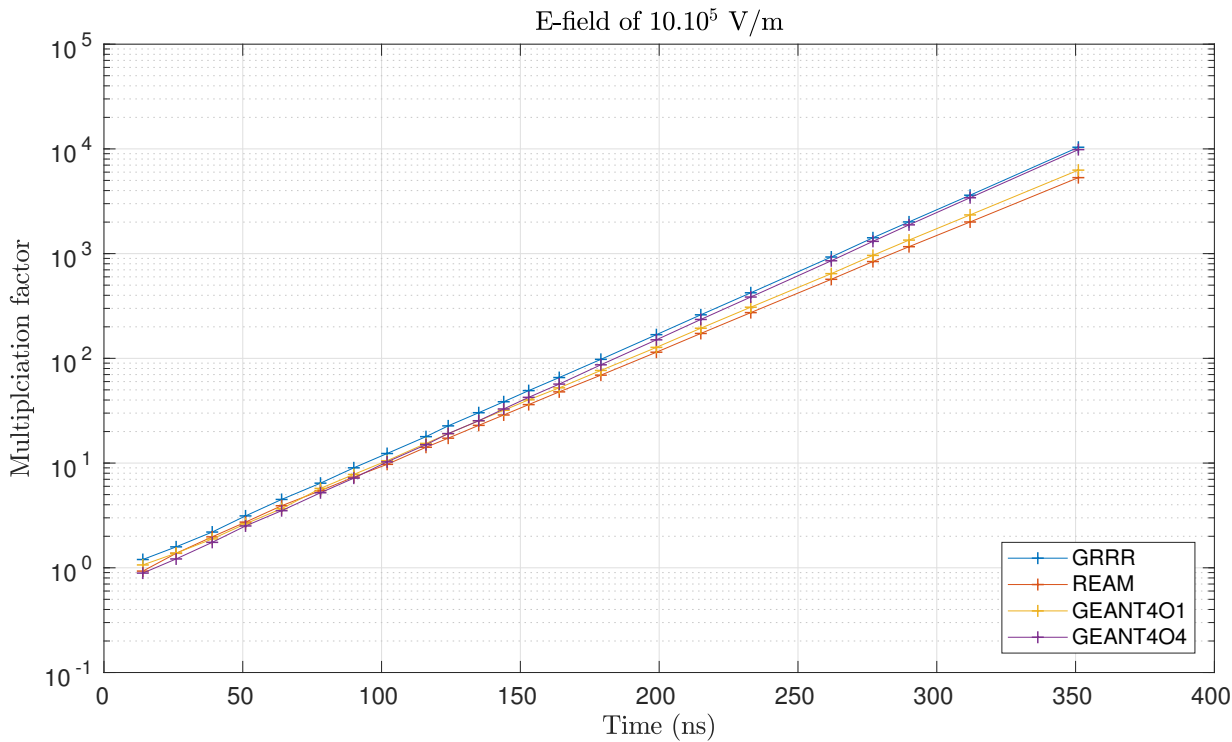
8.2 Avalanche Time And Length Code Comparison



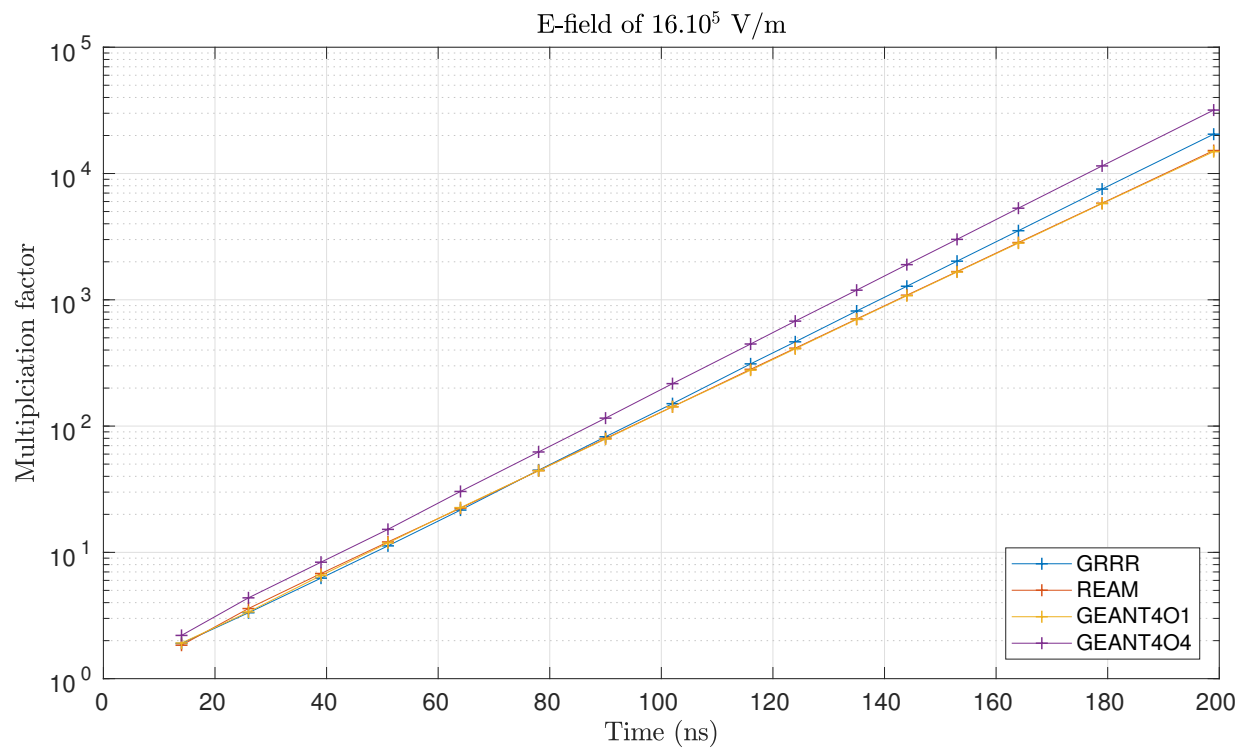
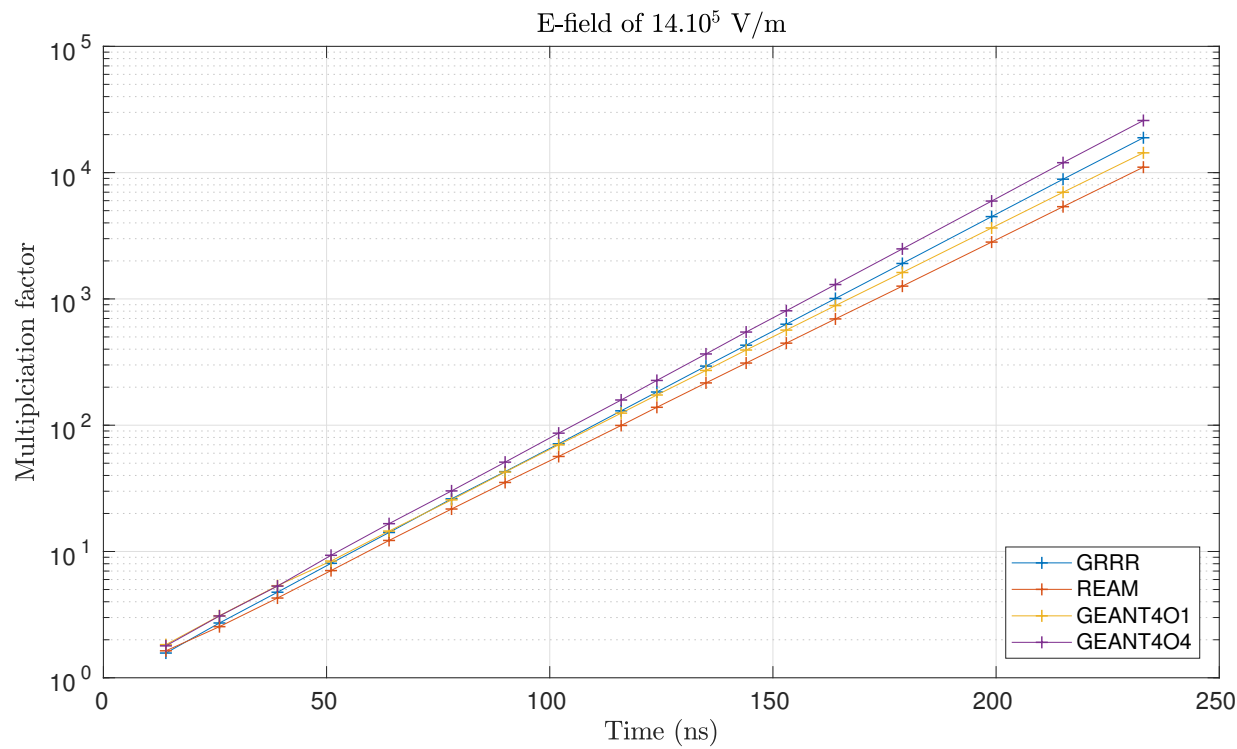


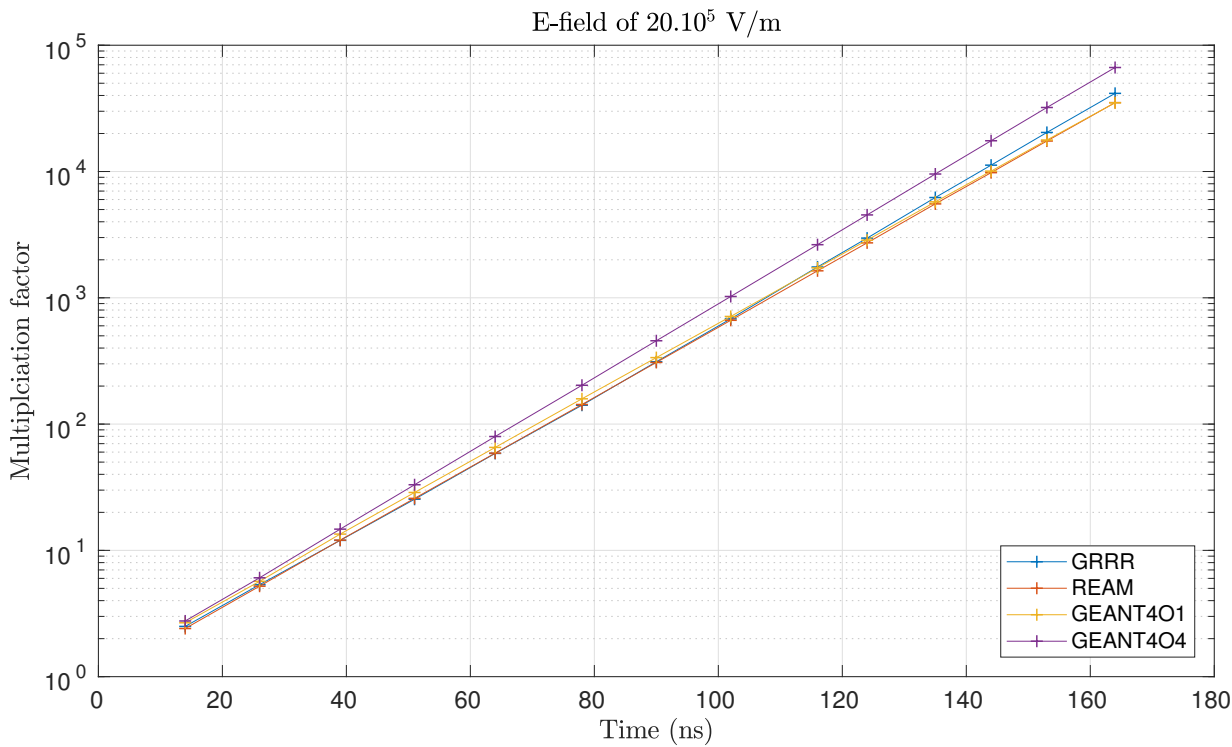
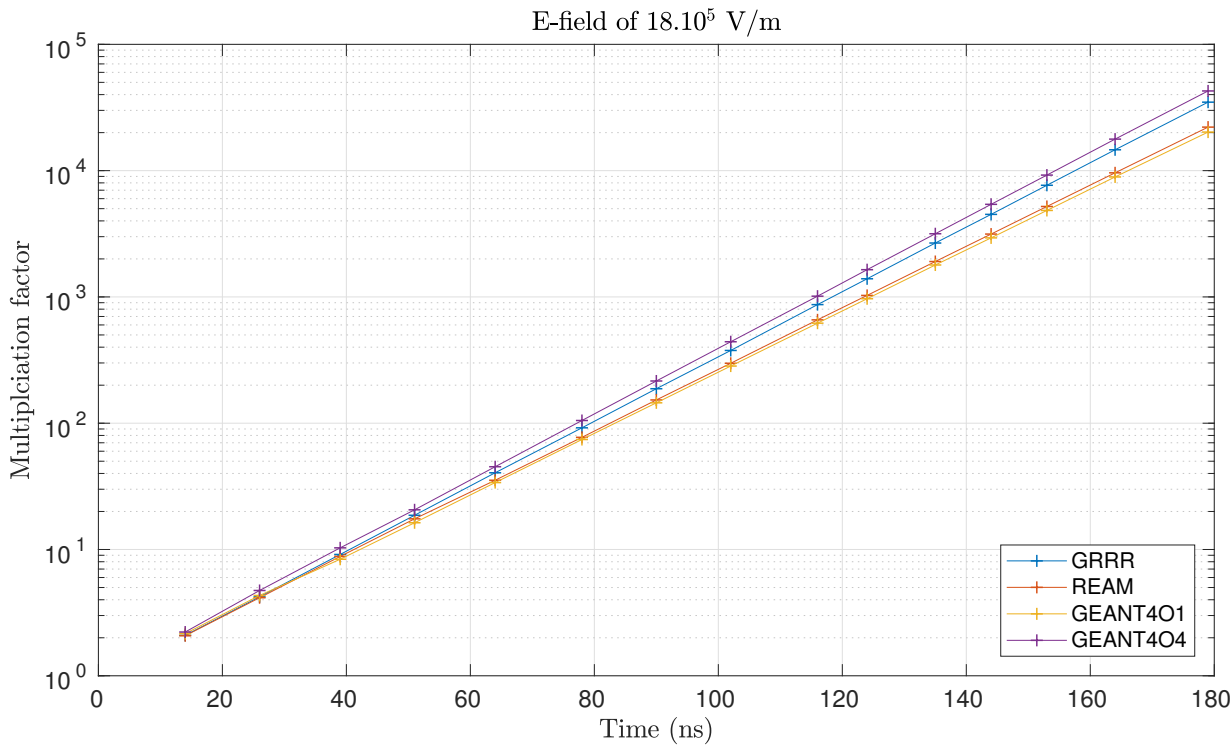
8.3 Time Record Code Comparison

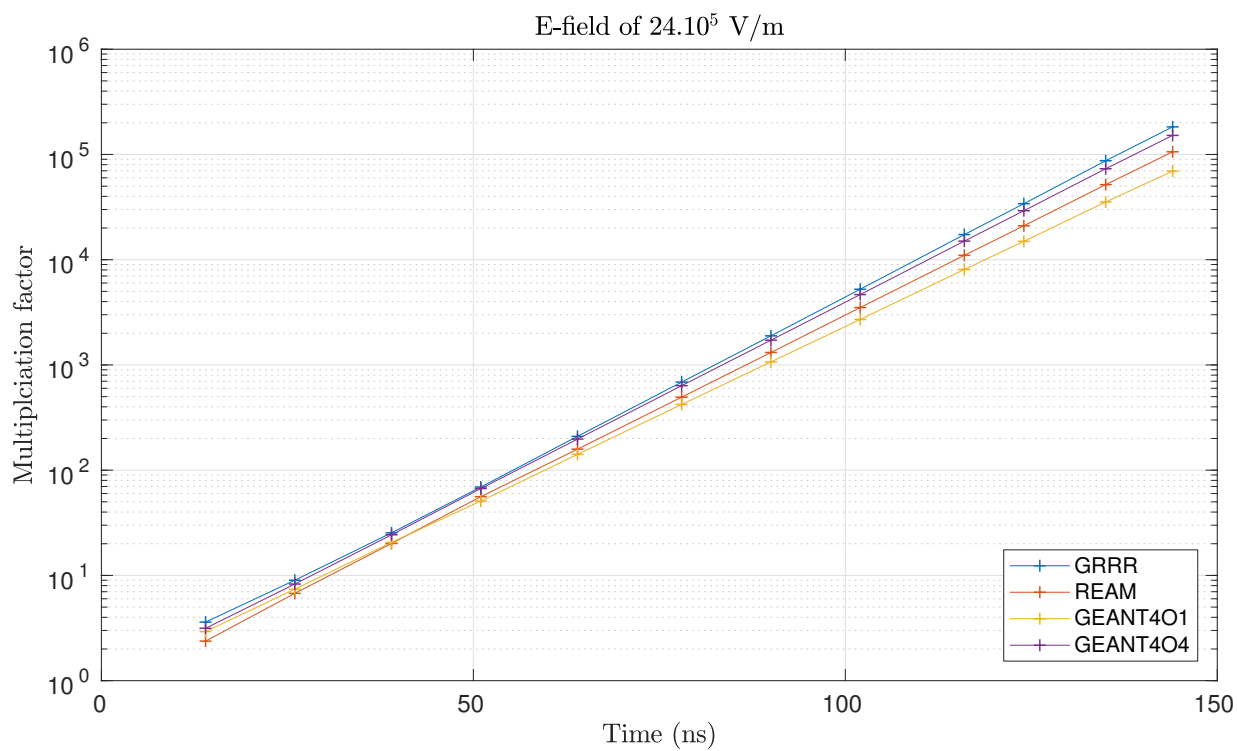
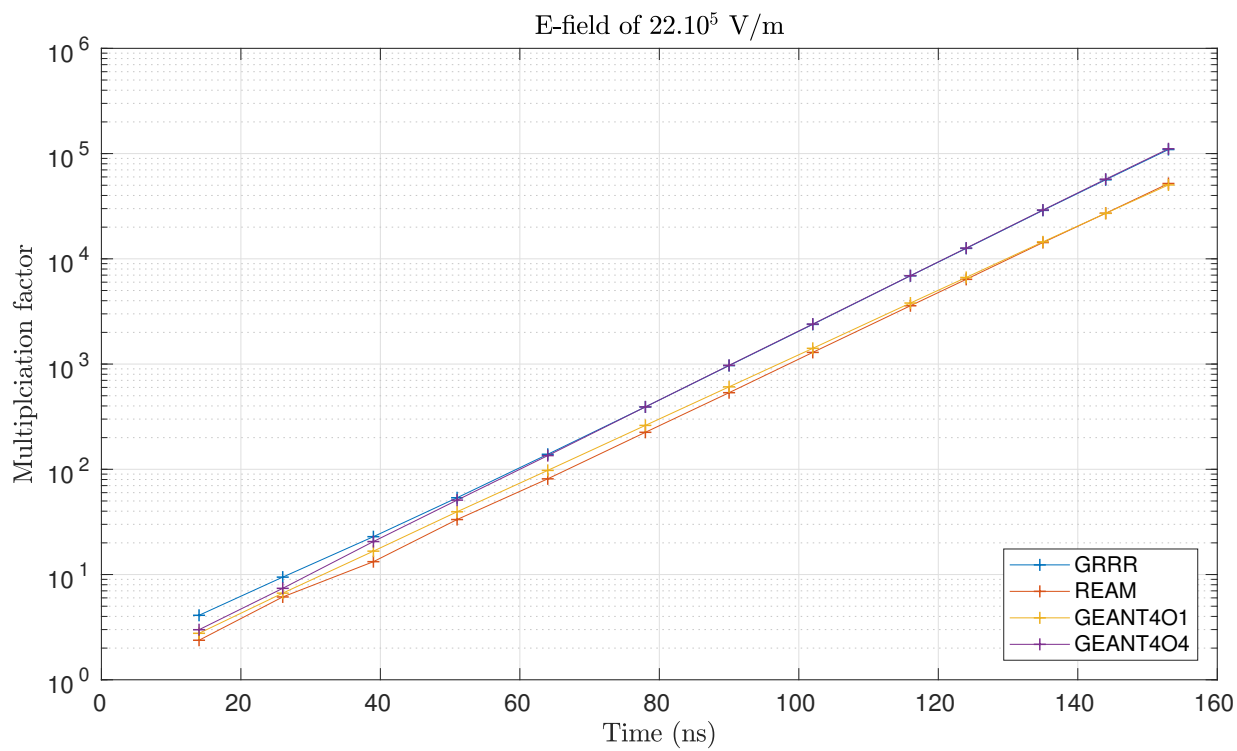


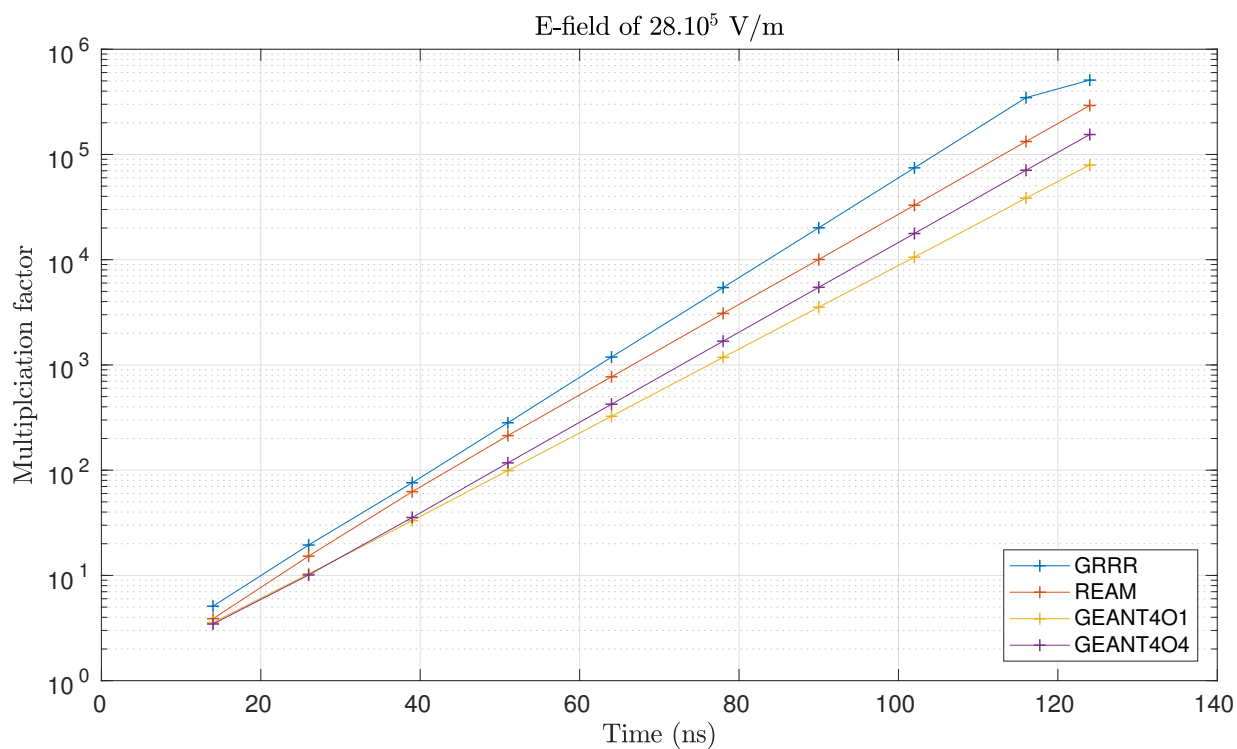
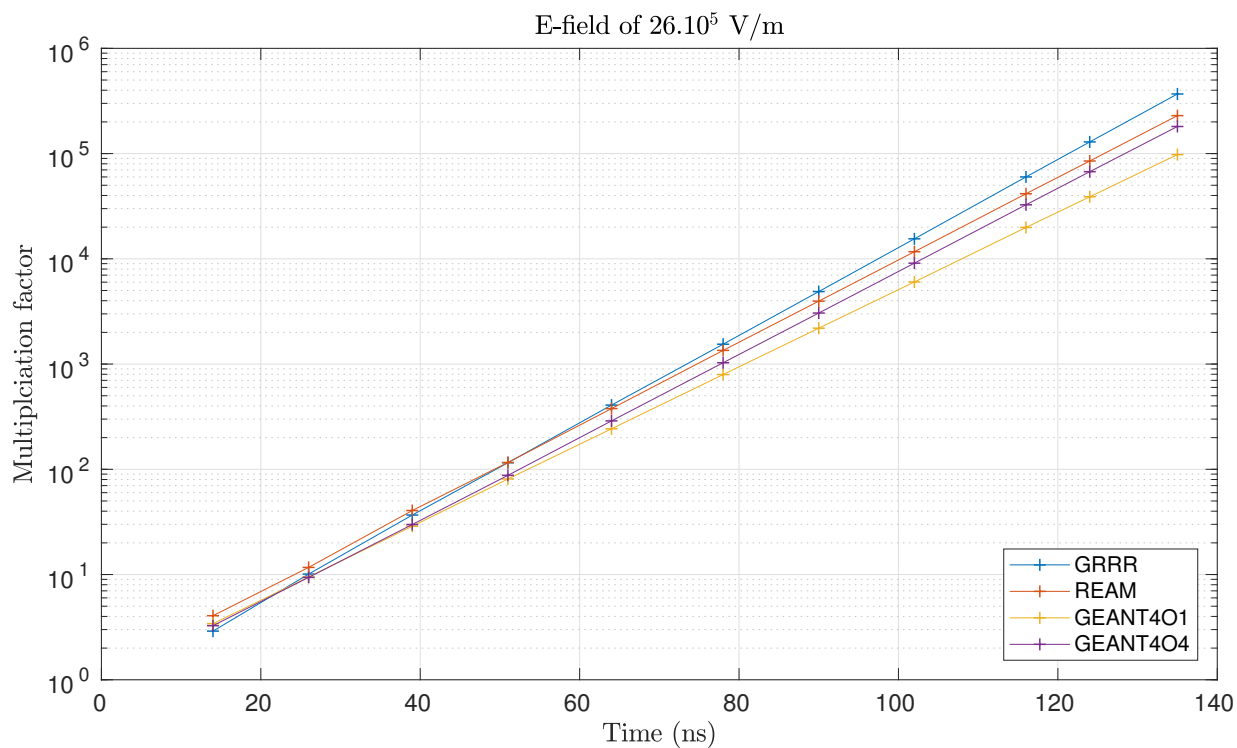


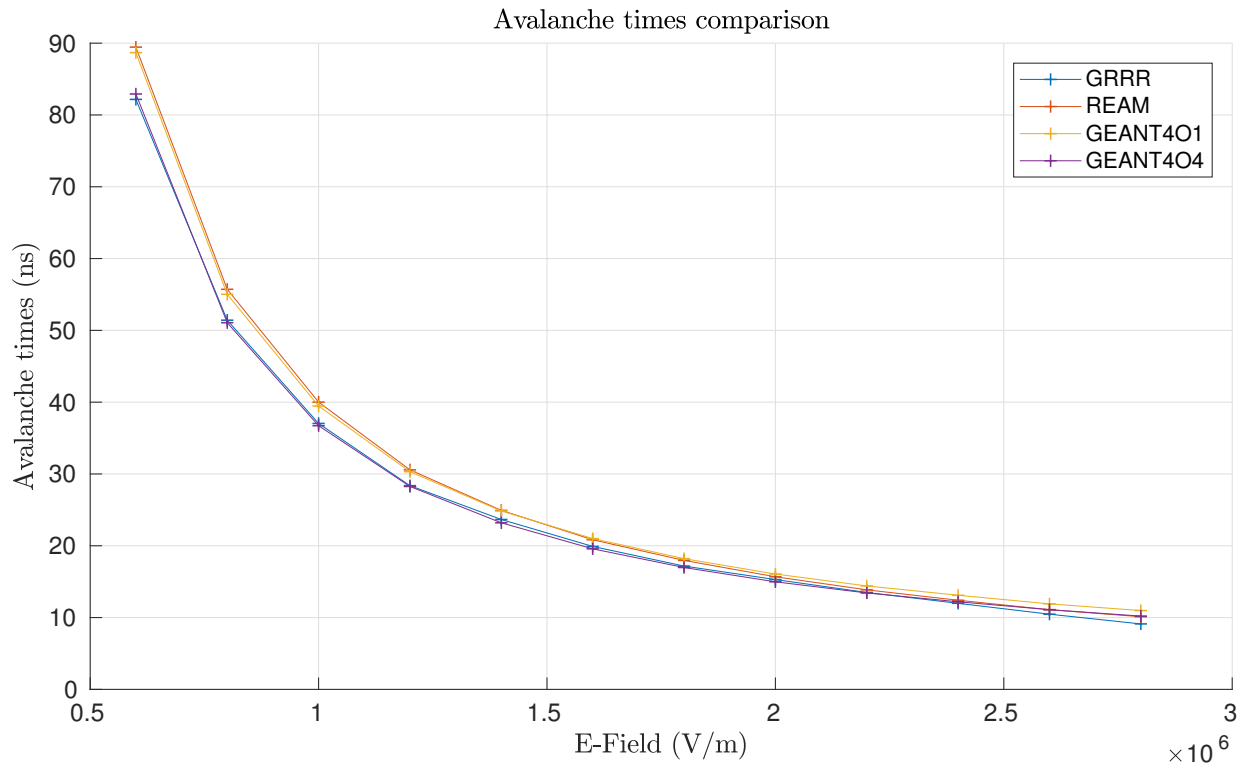




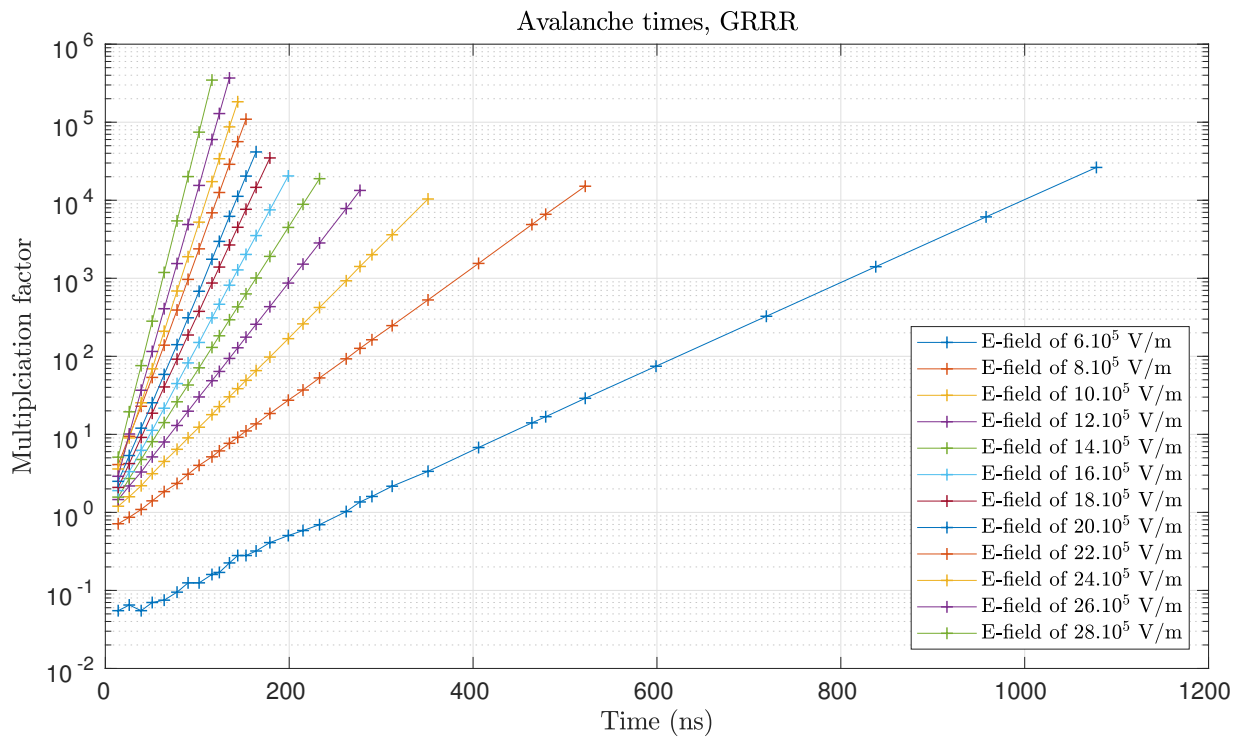


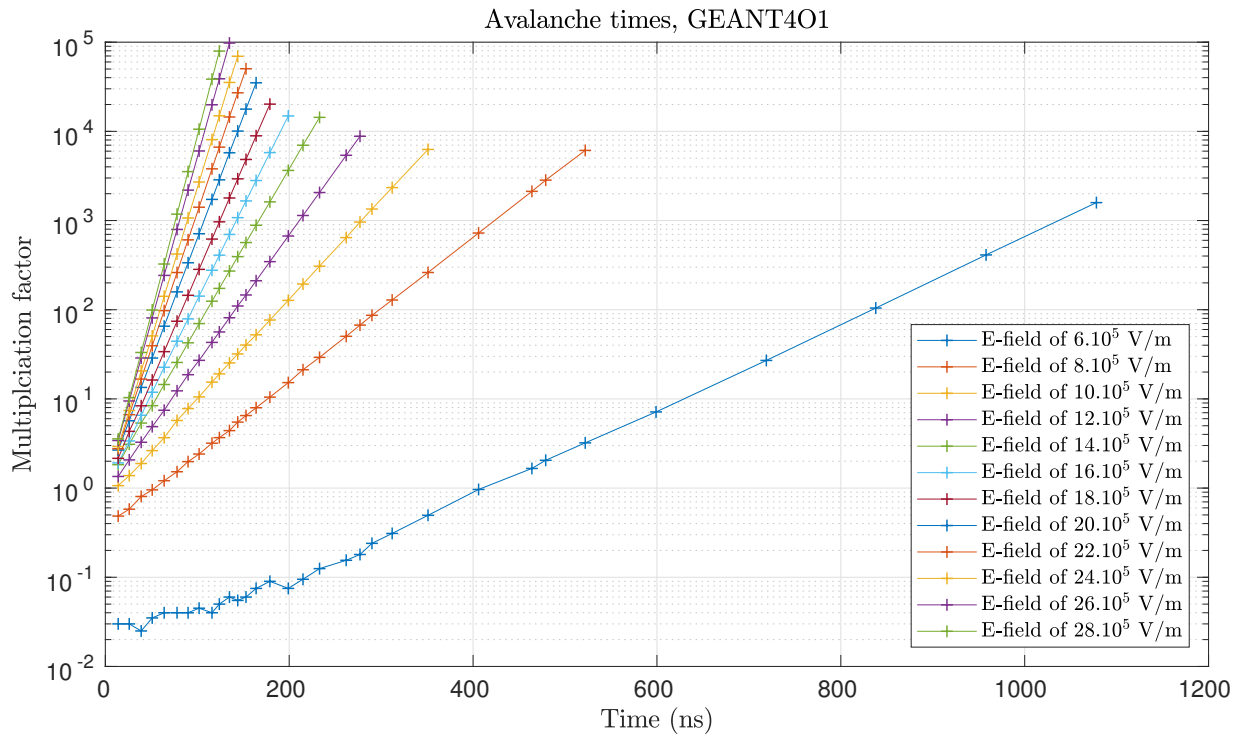
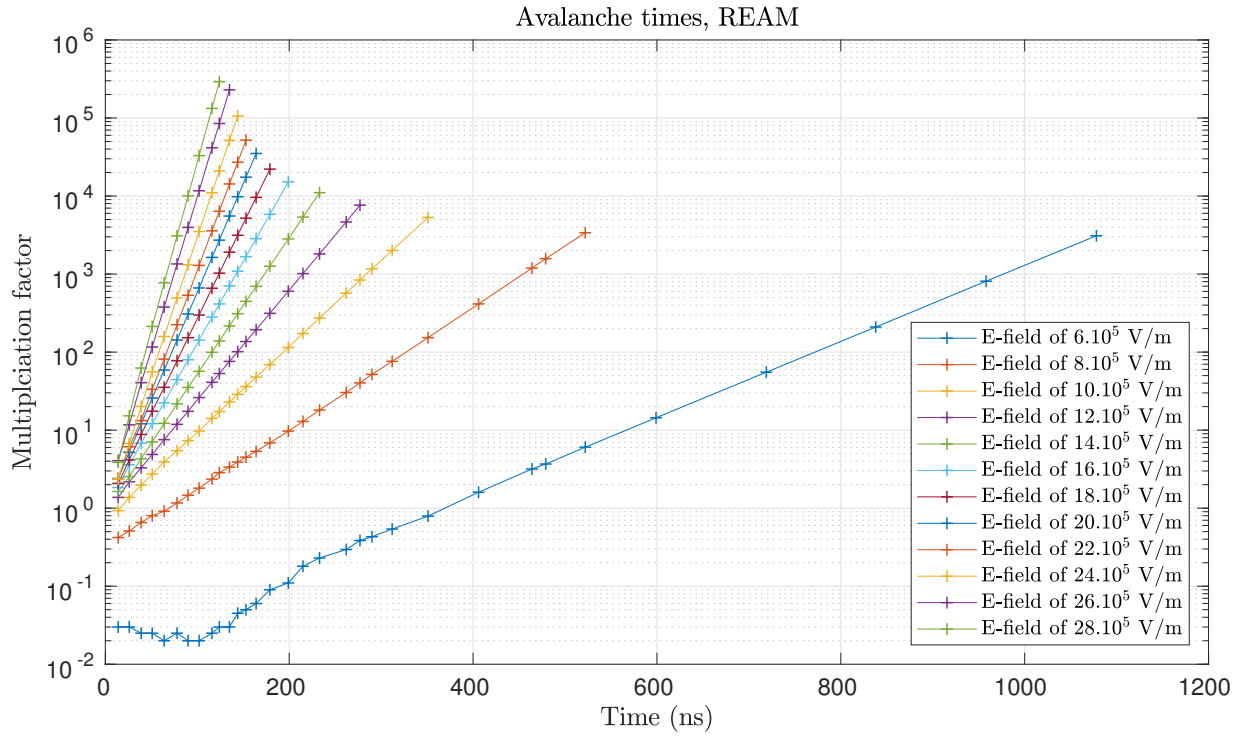


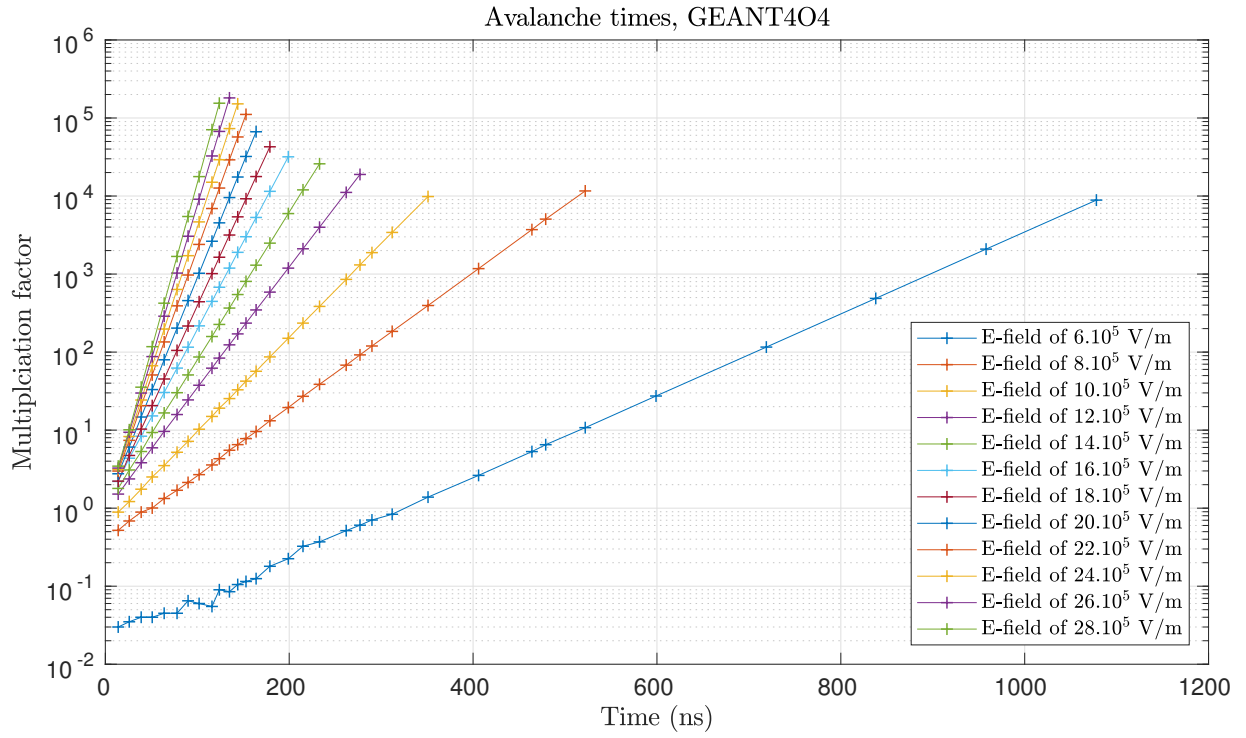




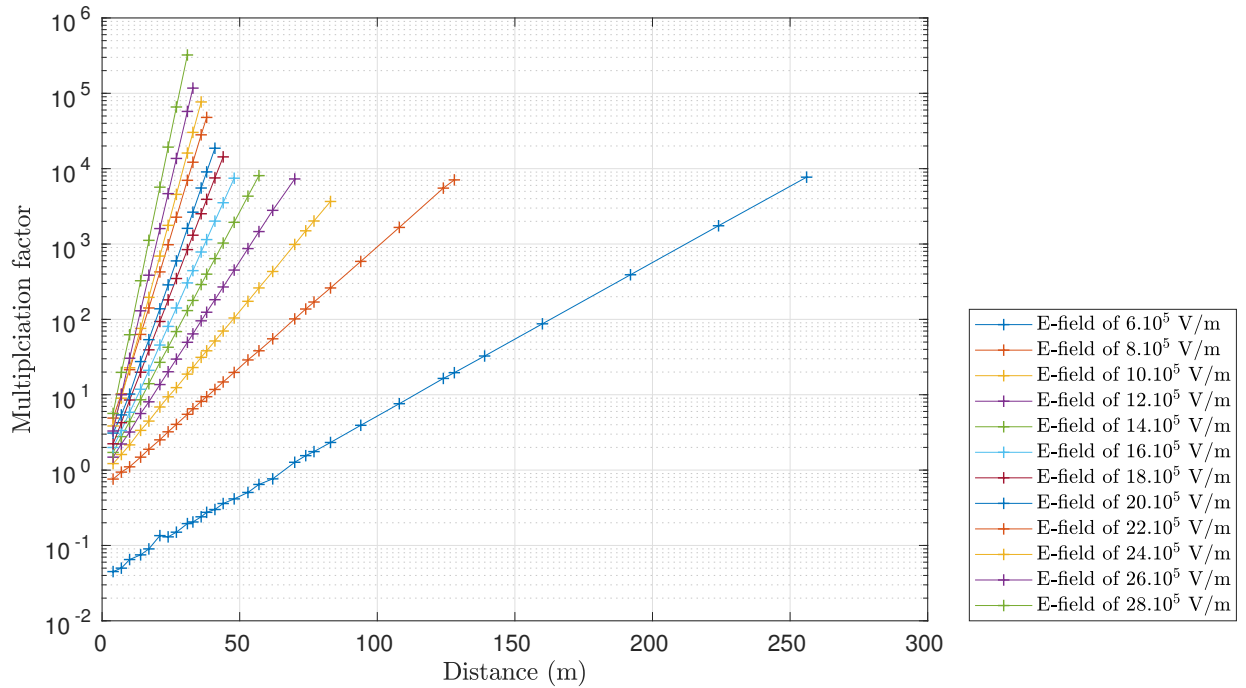
#### 8.4 Time Record Code By Code

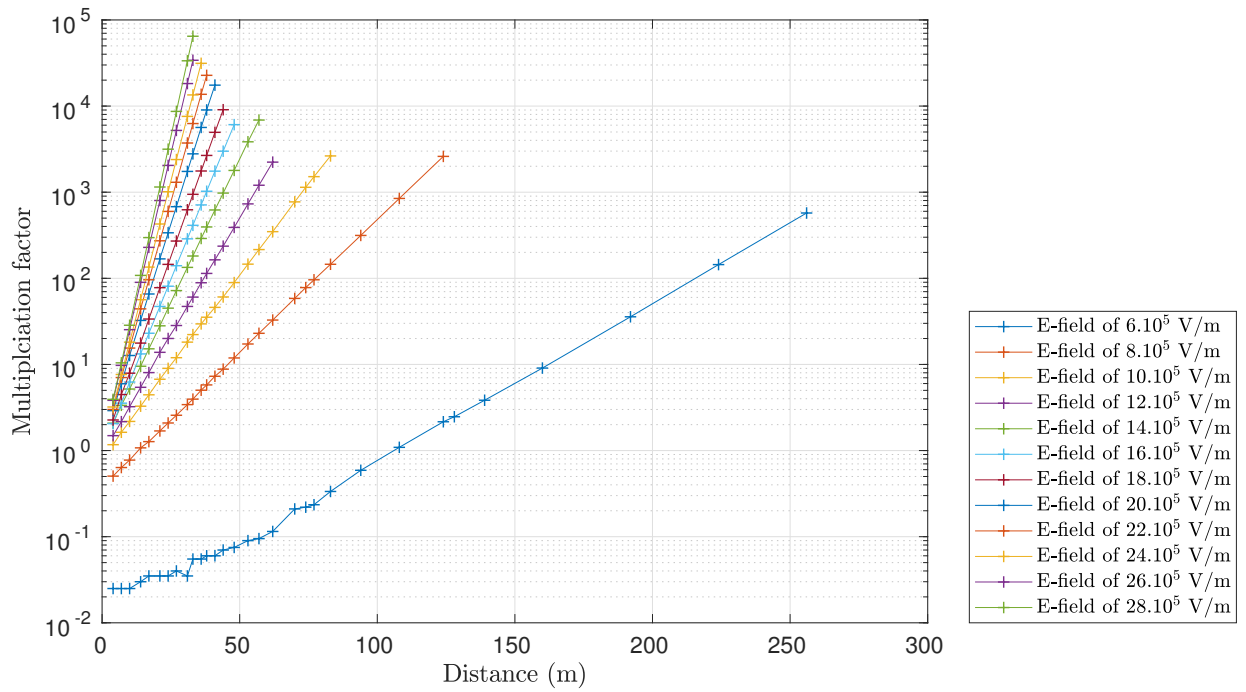
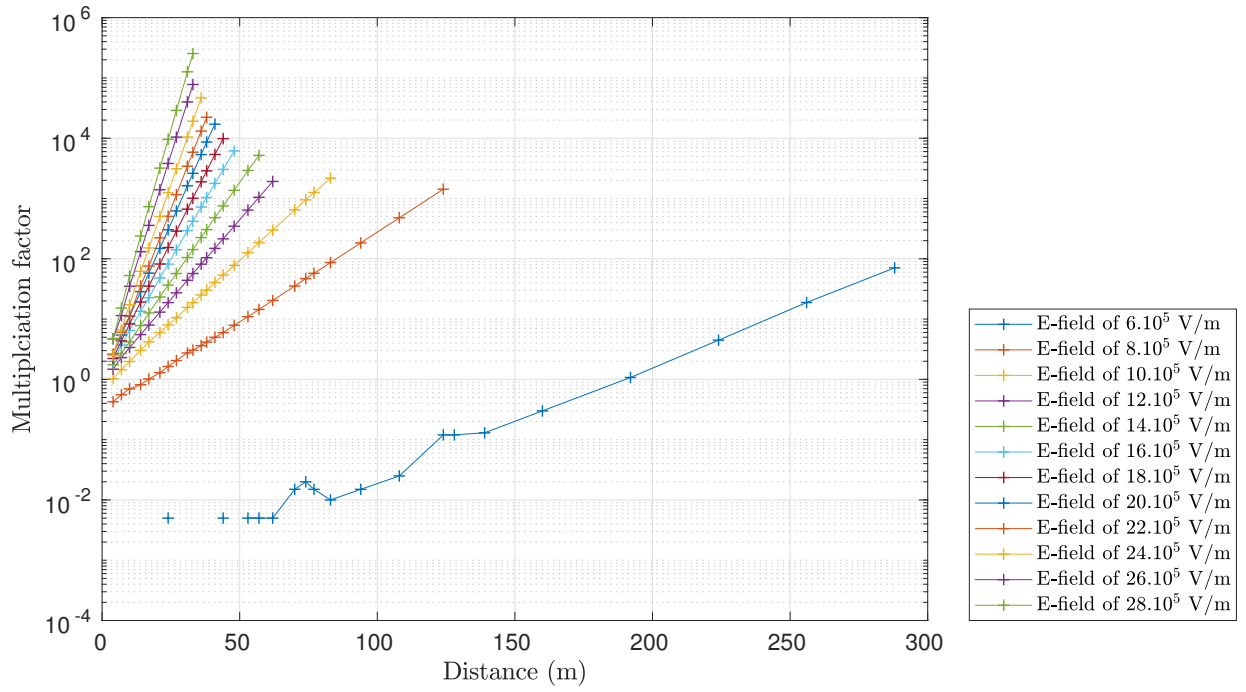




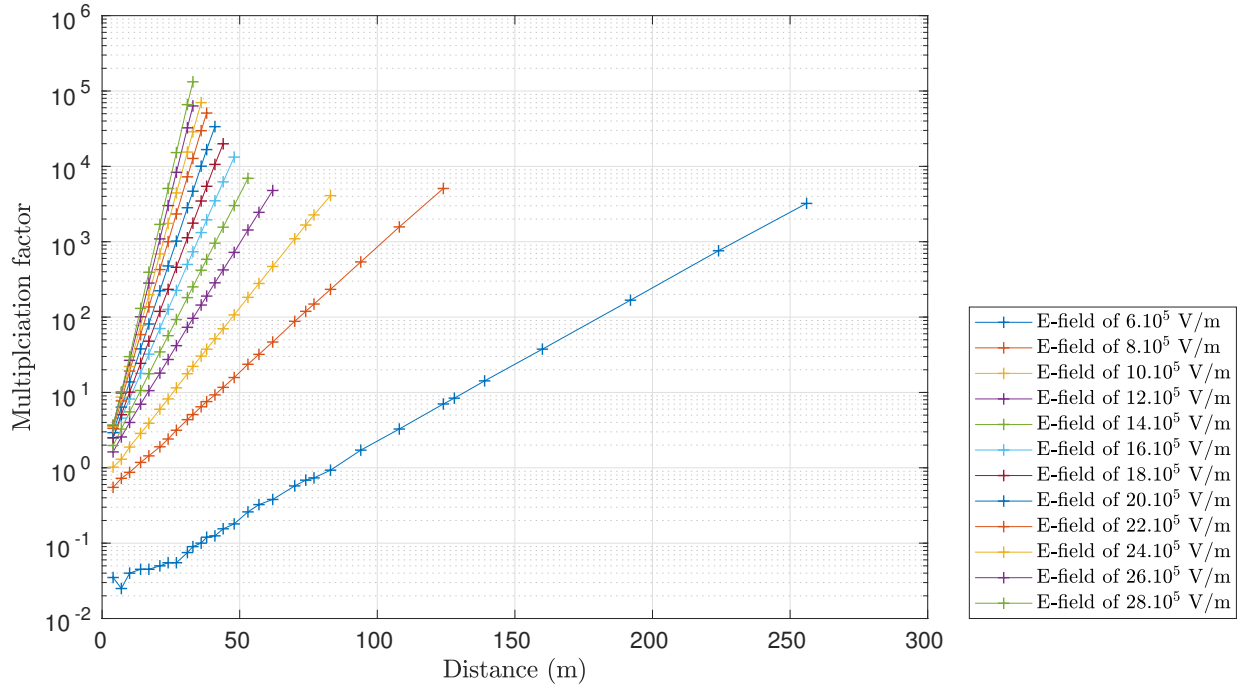


### 8.5 Distance Record Code By Code

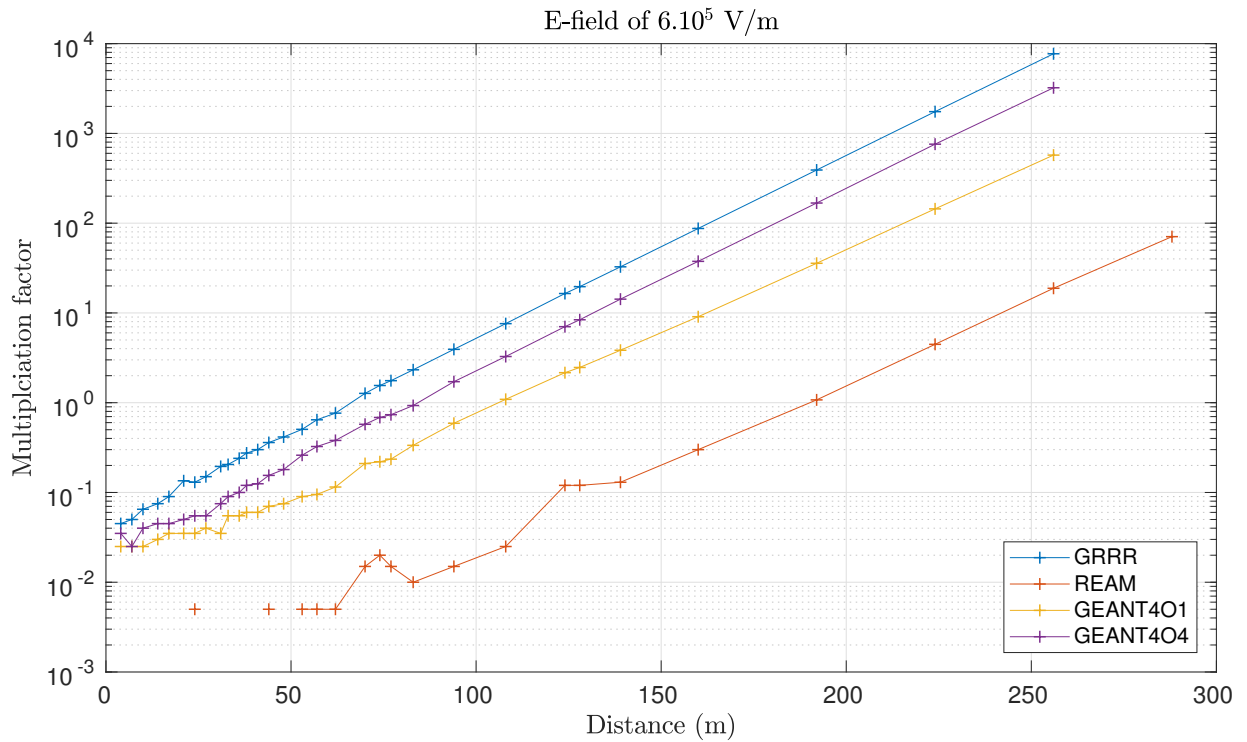


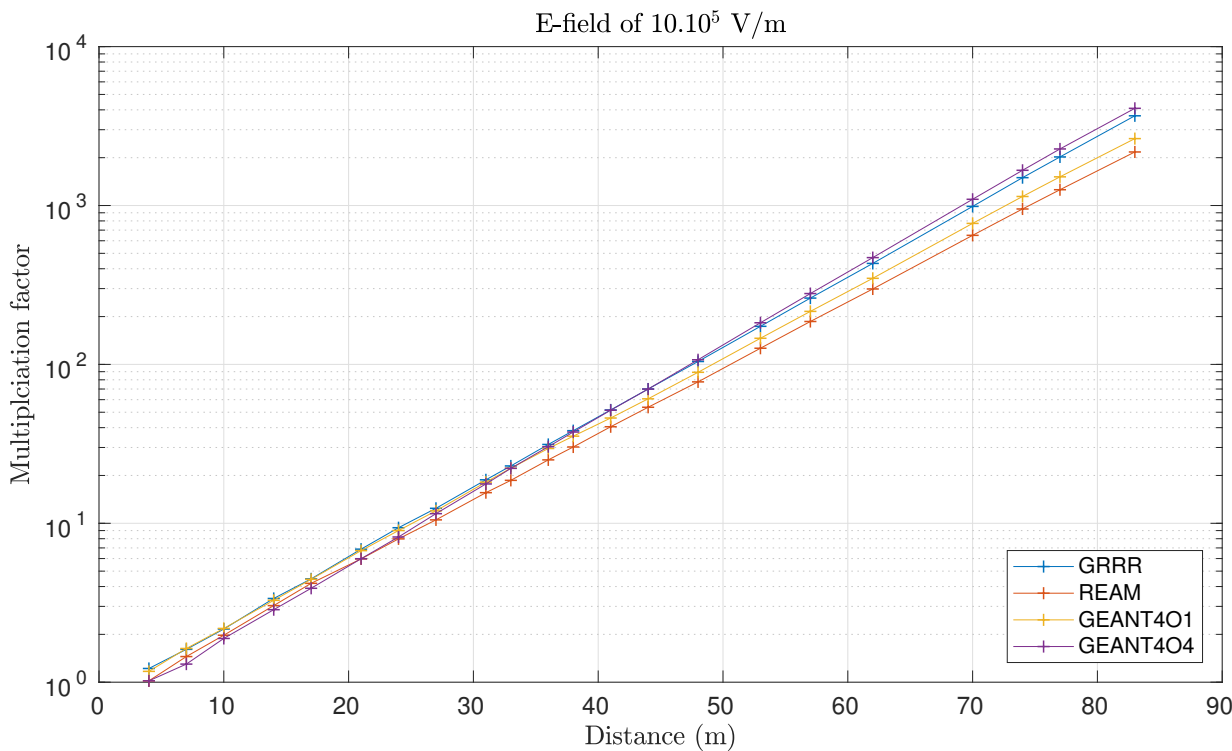
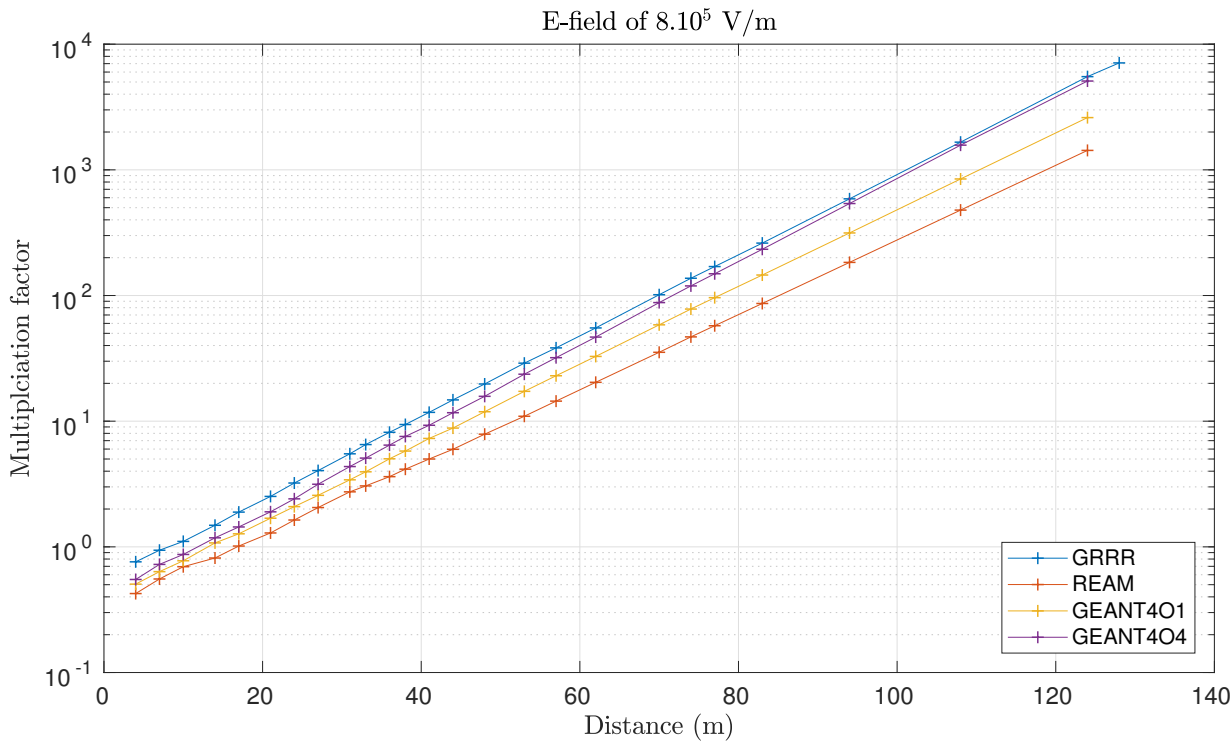


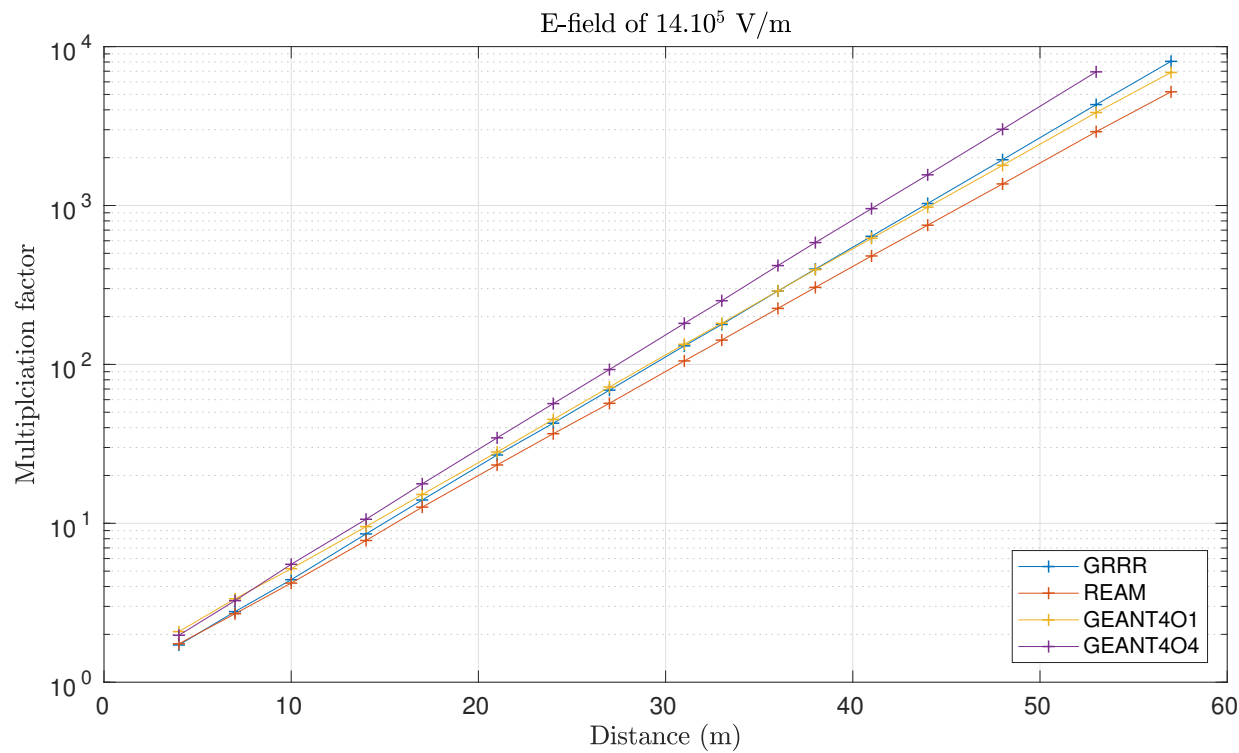
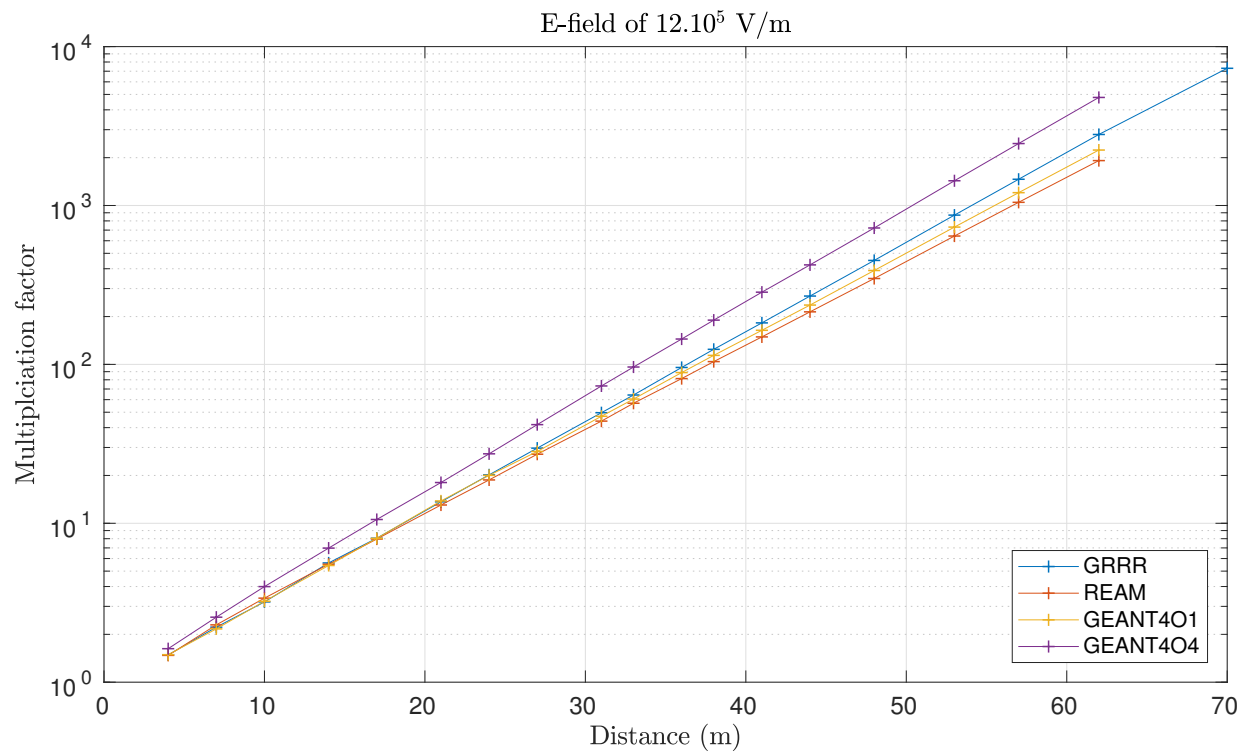


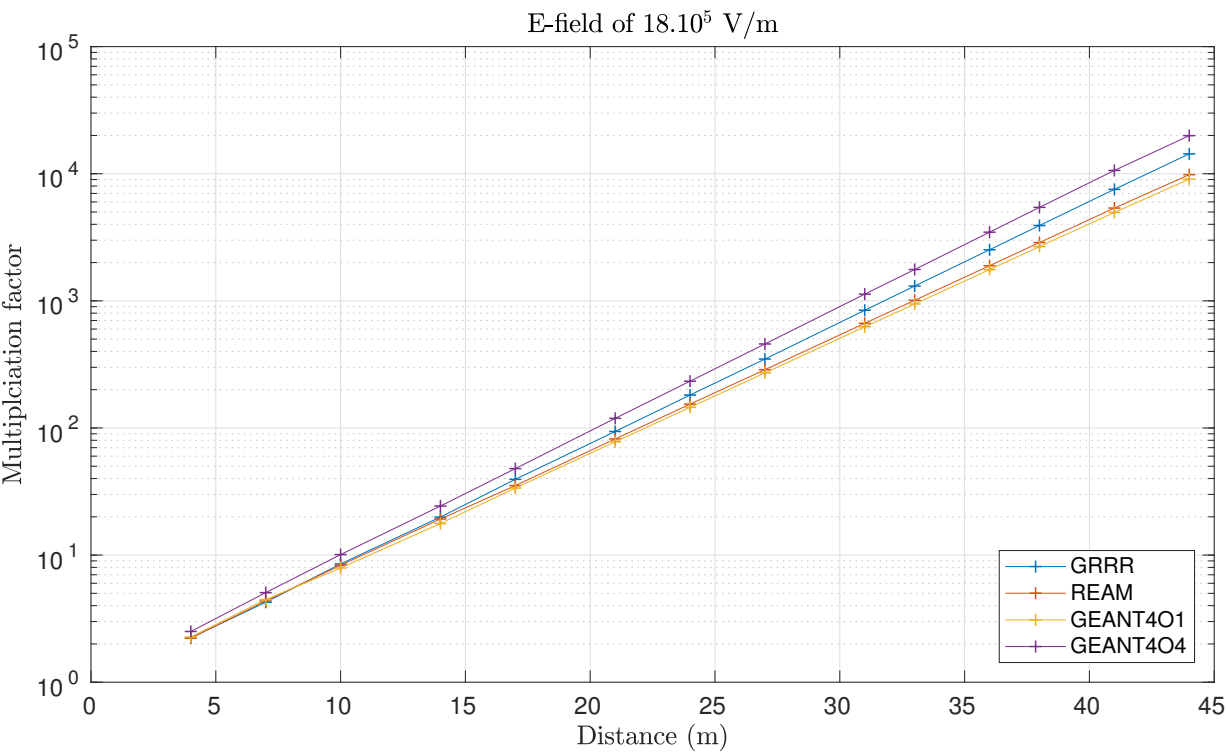
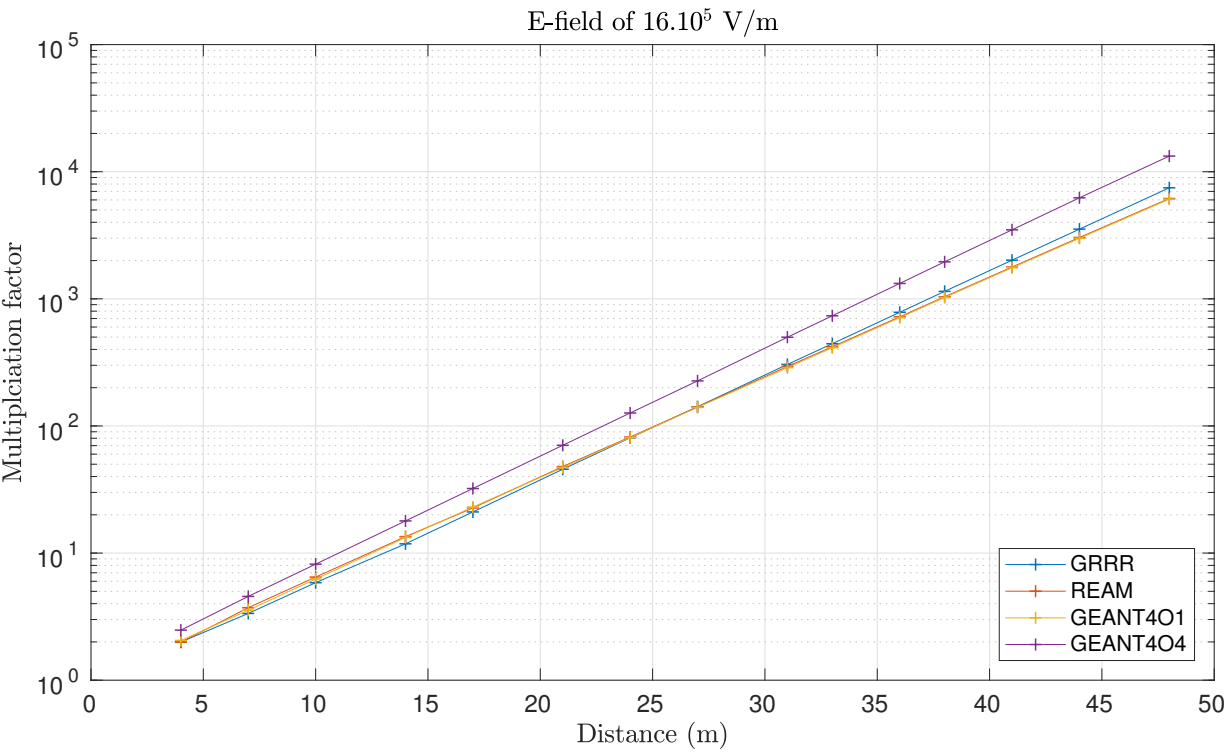


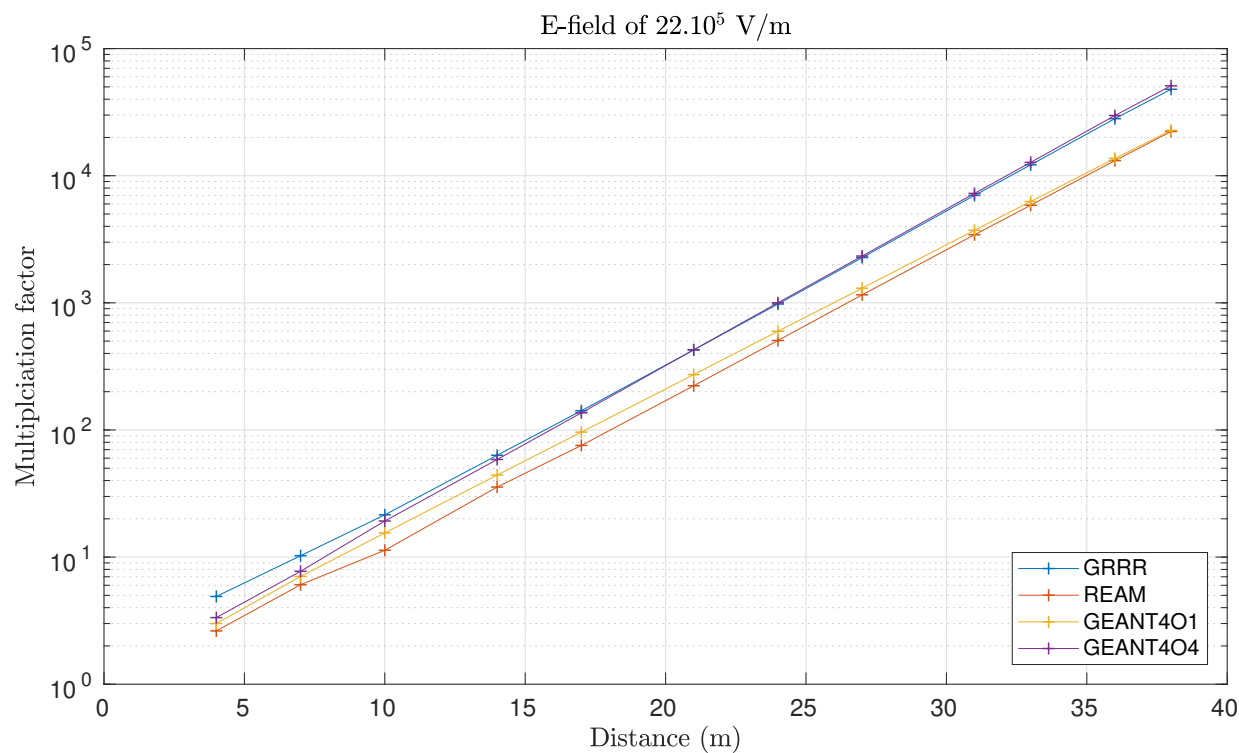
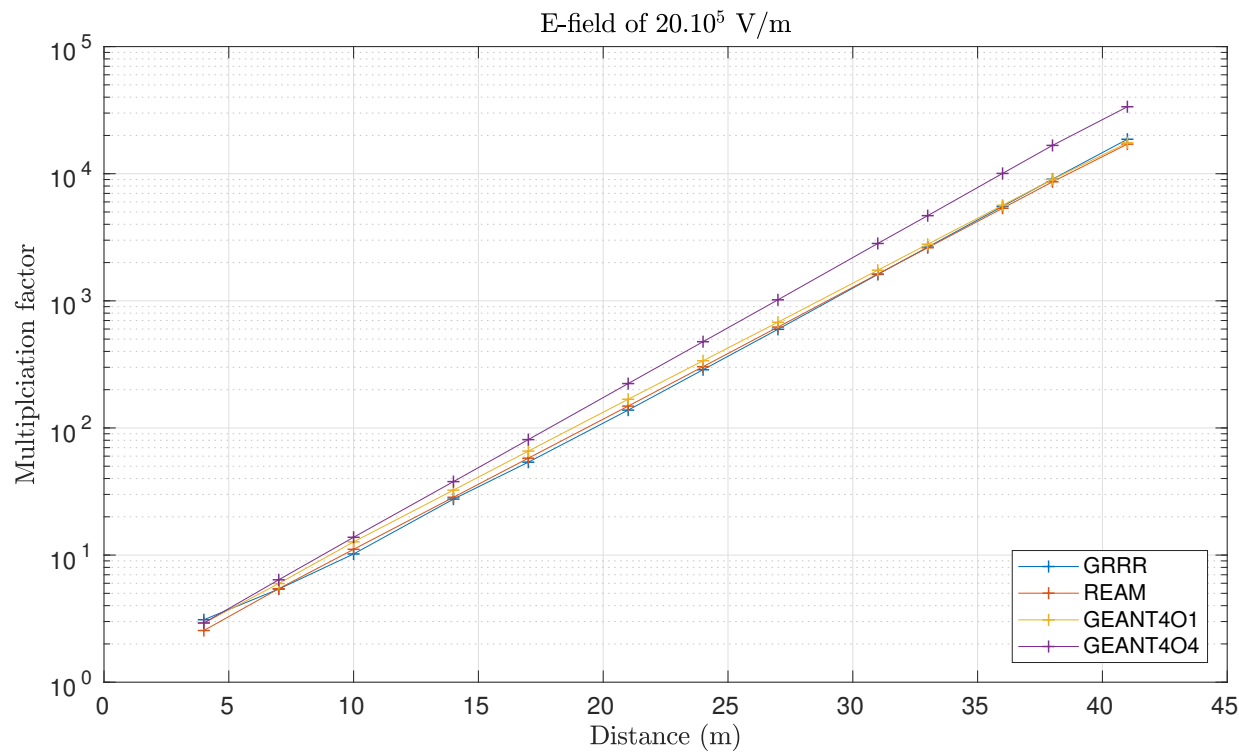
## 8.6 Distance Record Code Comparison

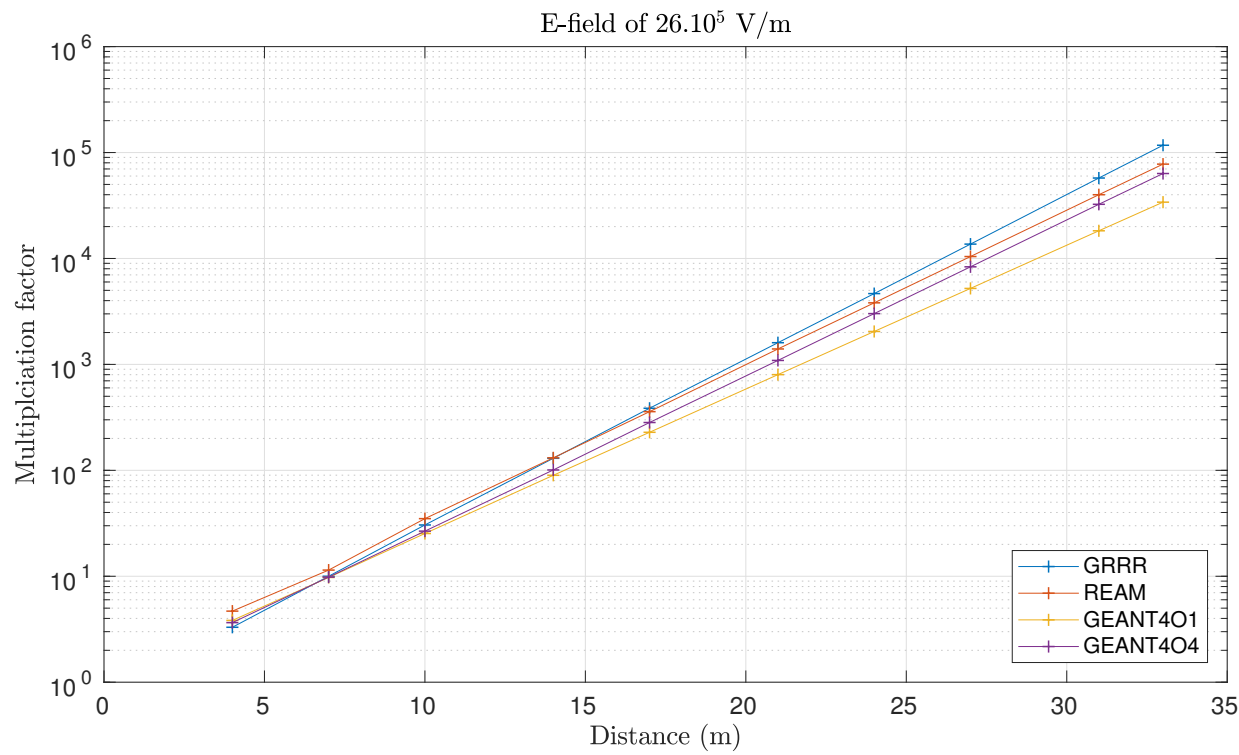
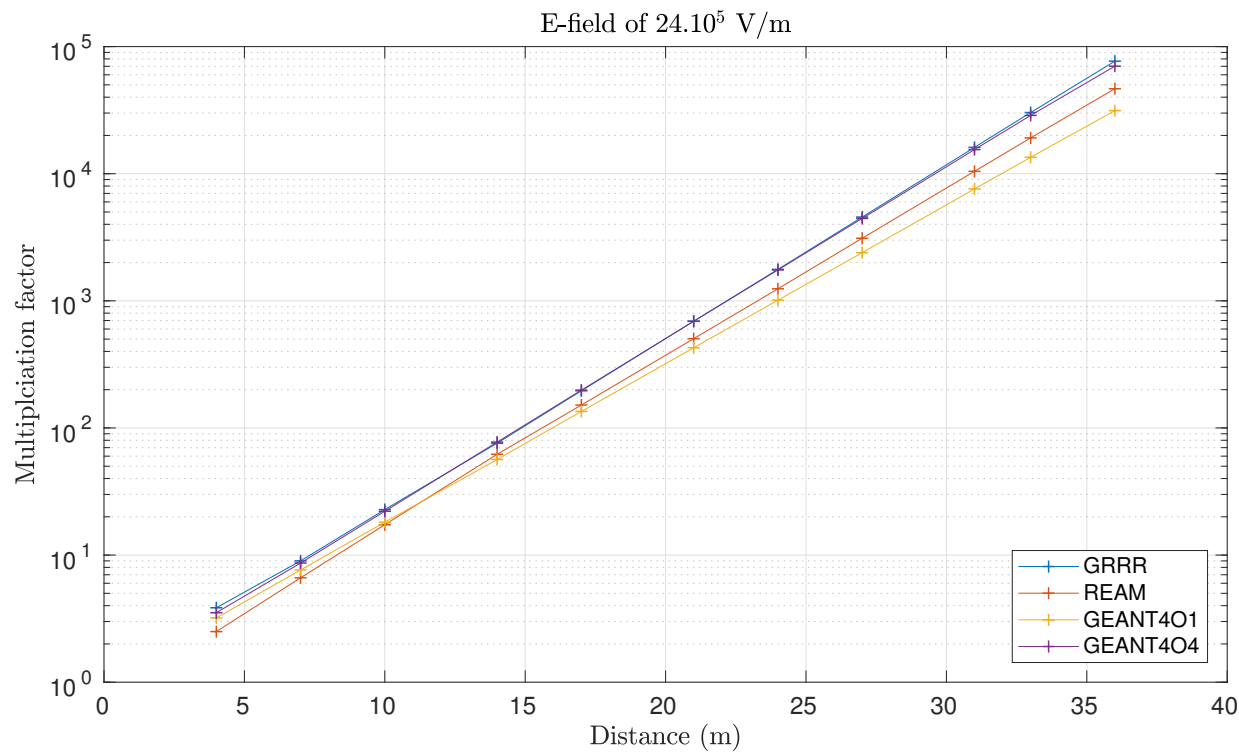


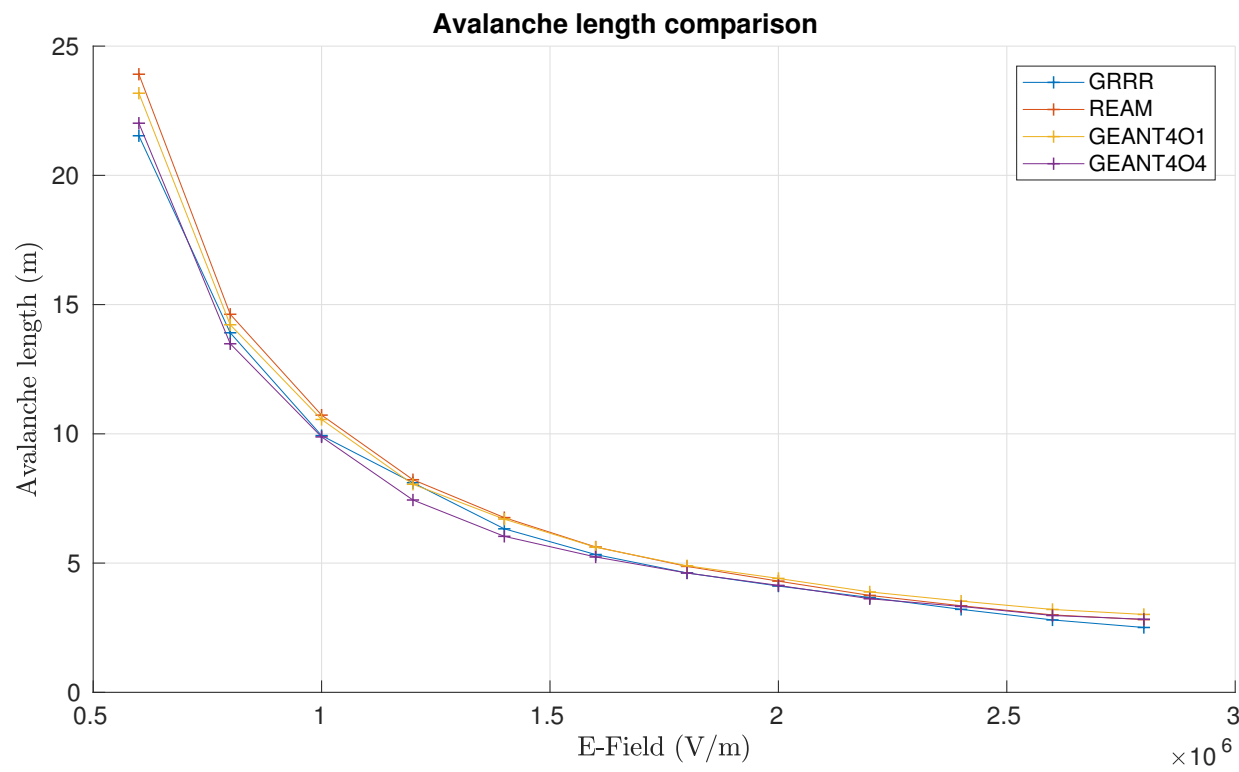
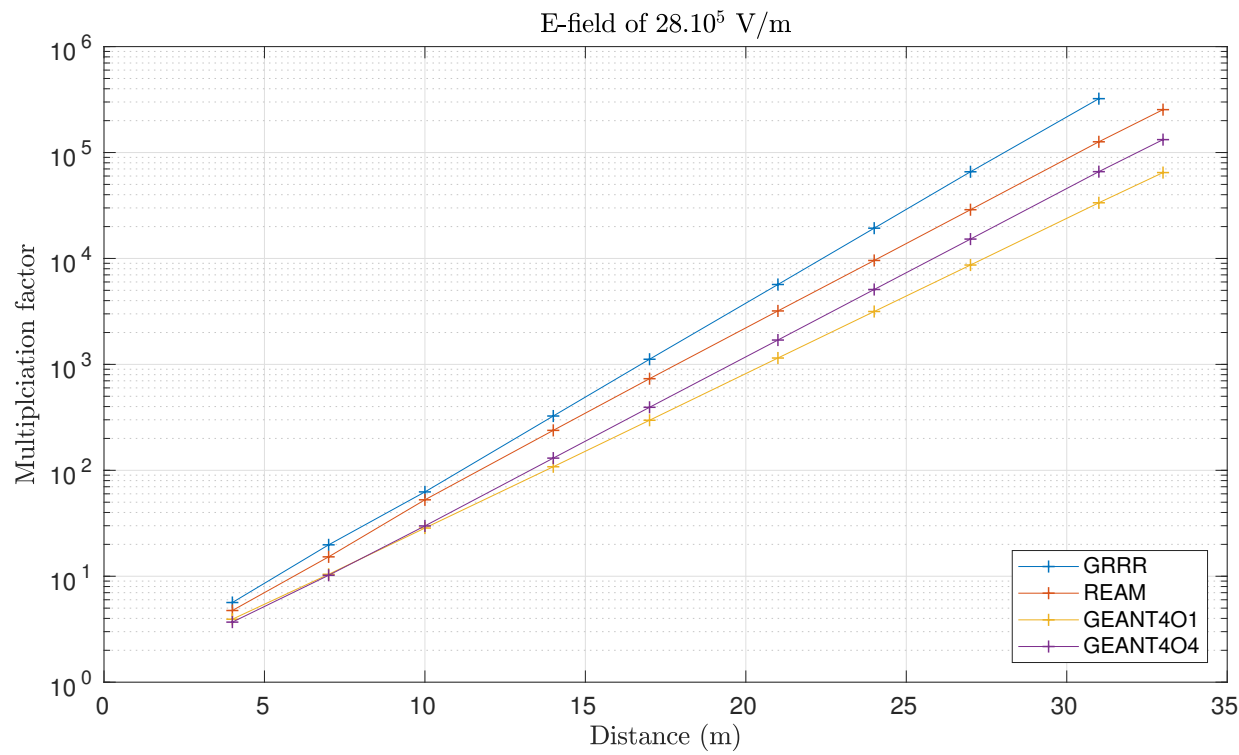






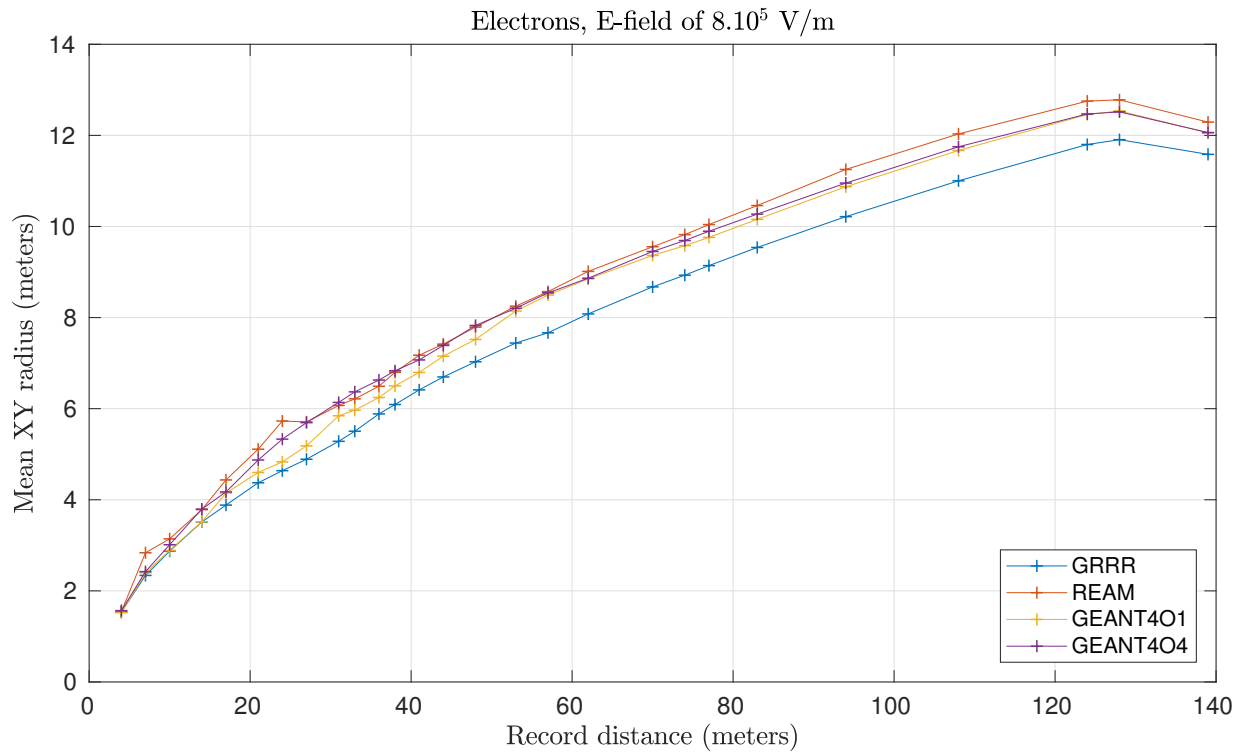
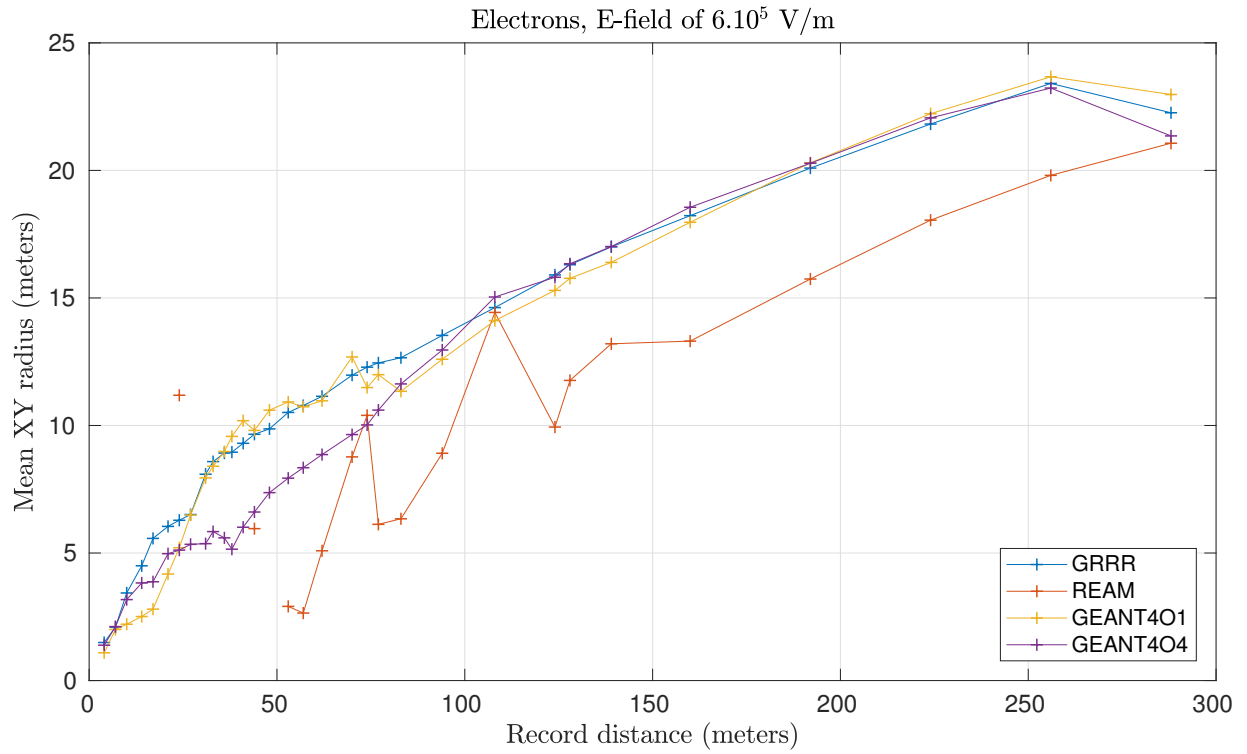




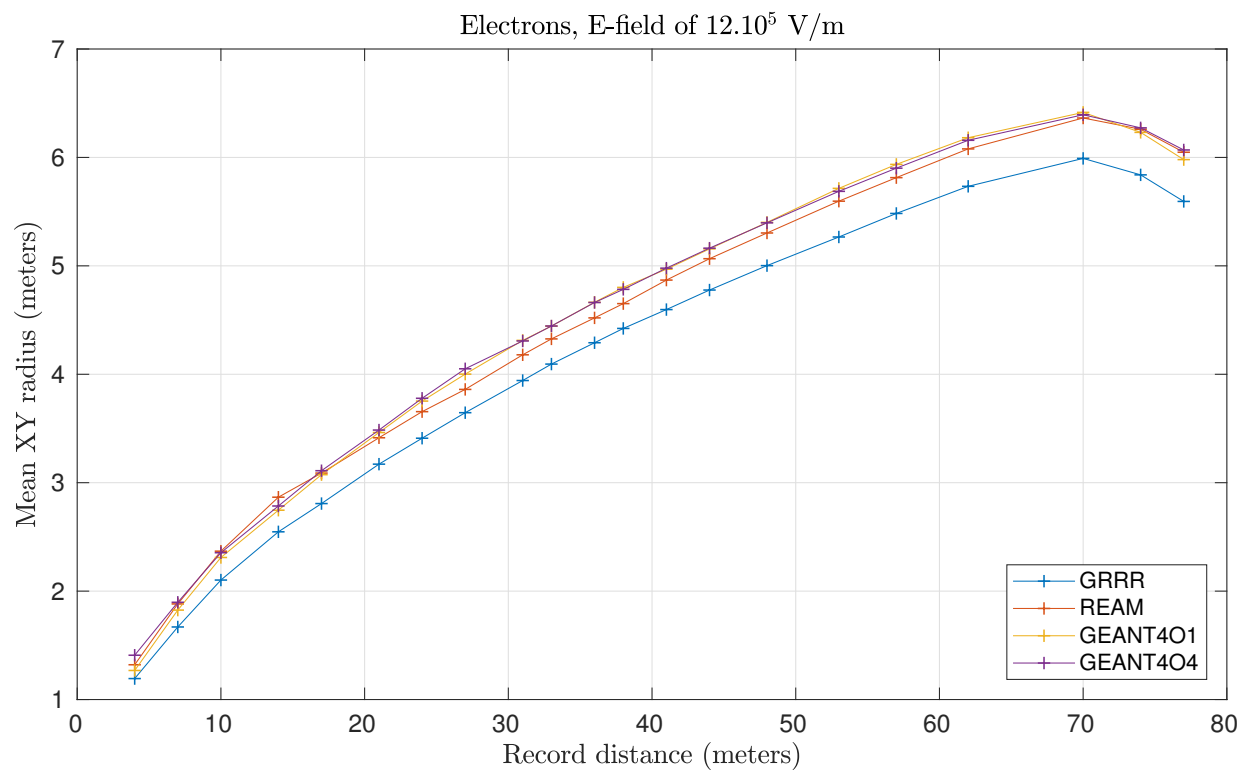
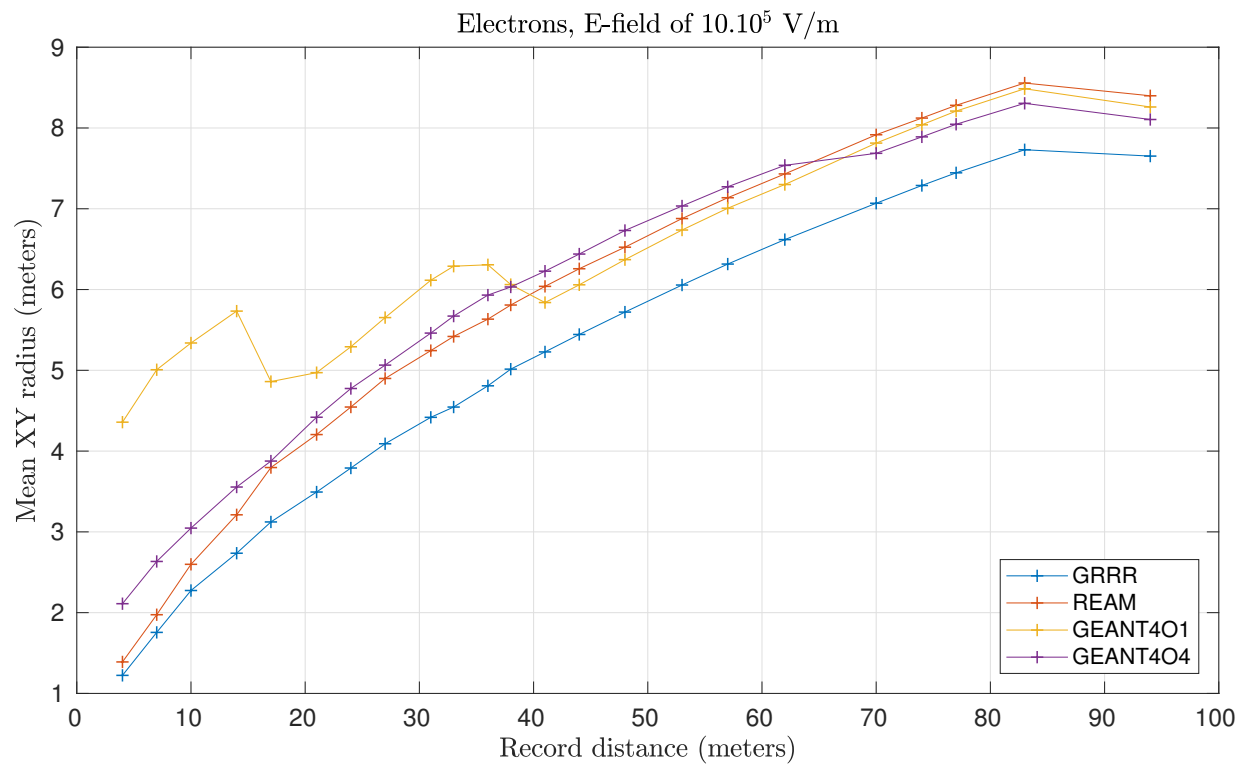


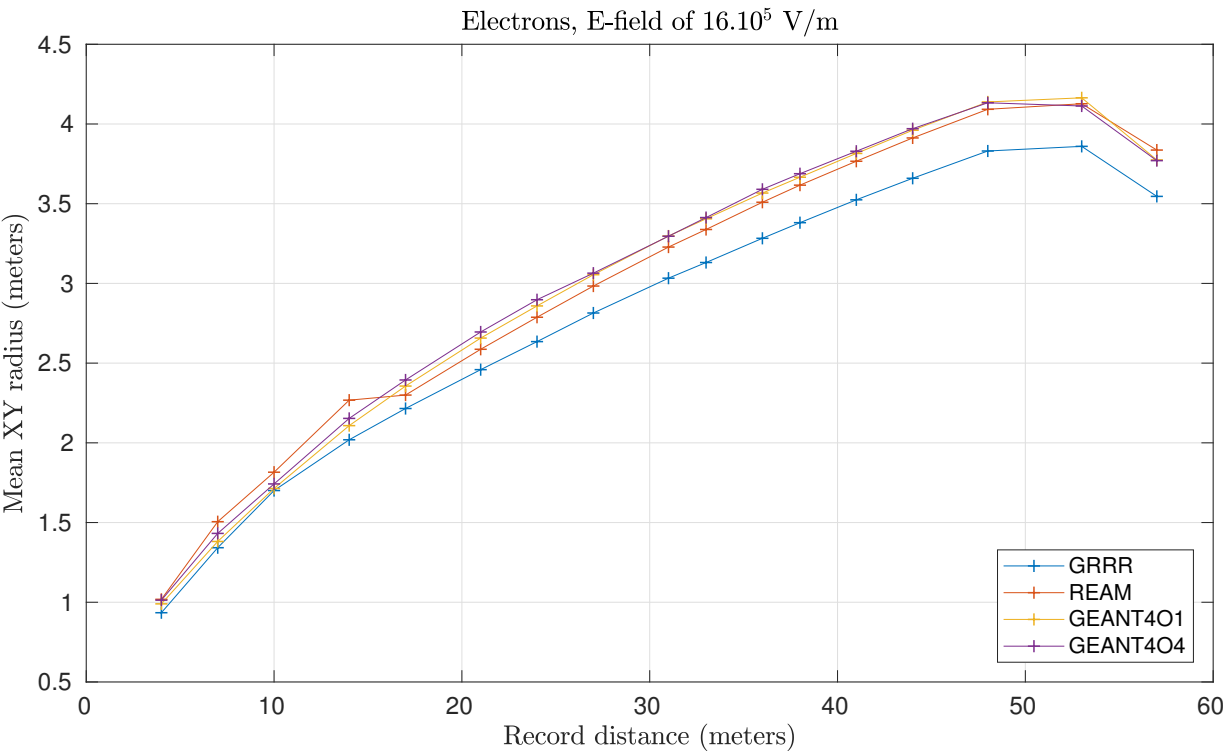
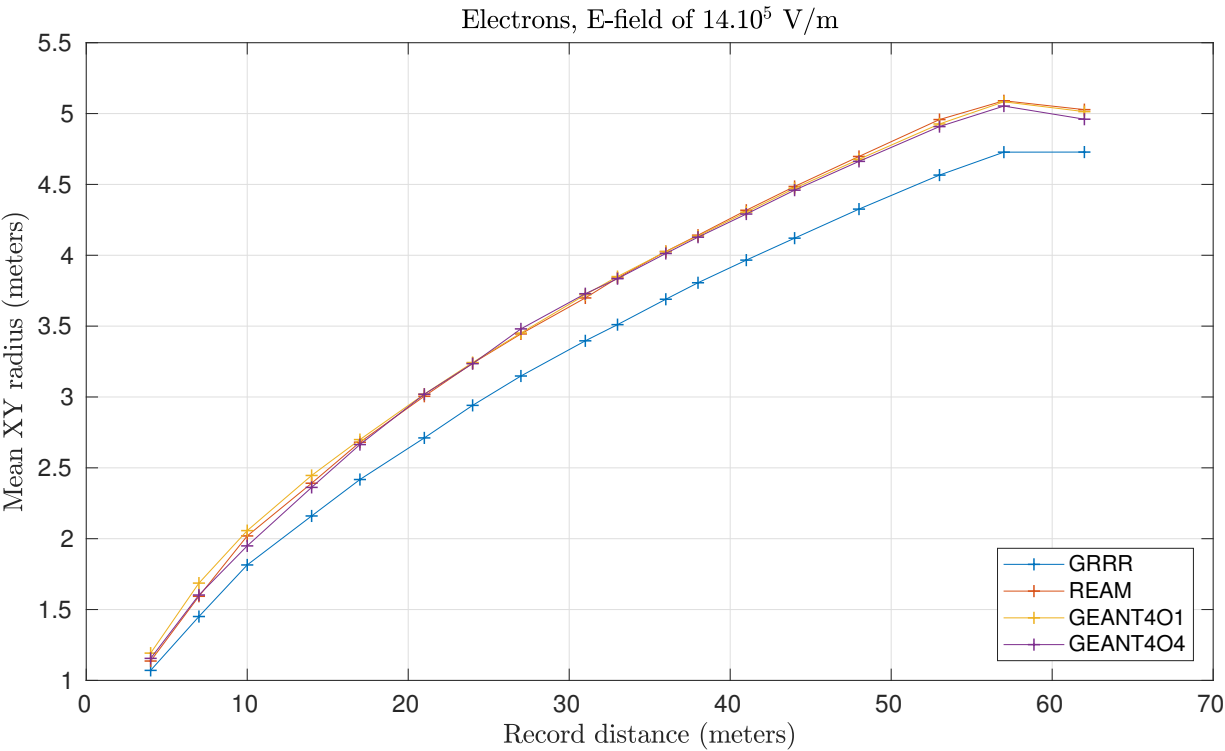
## 9 Mean XY Radius

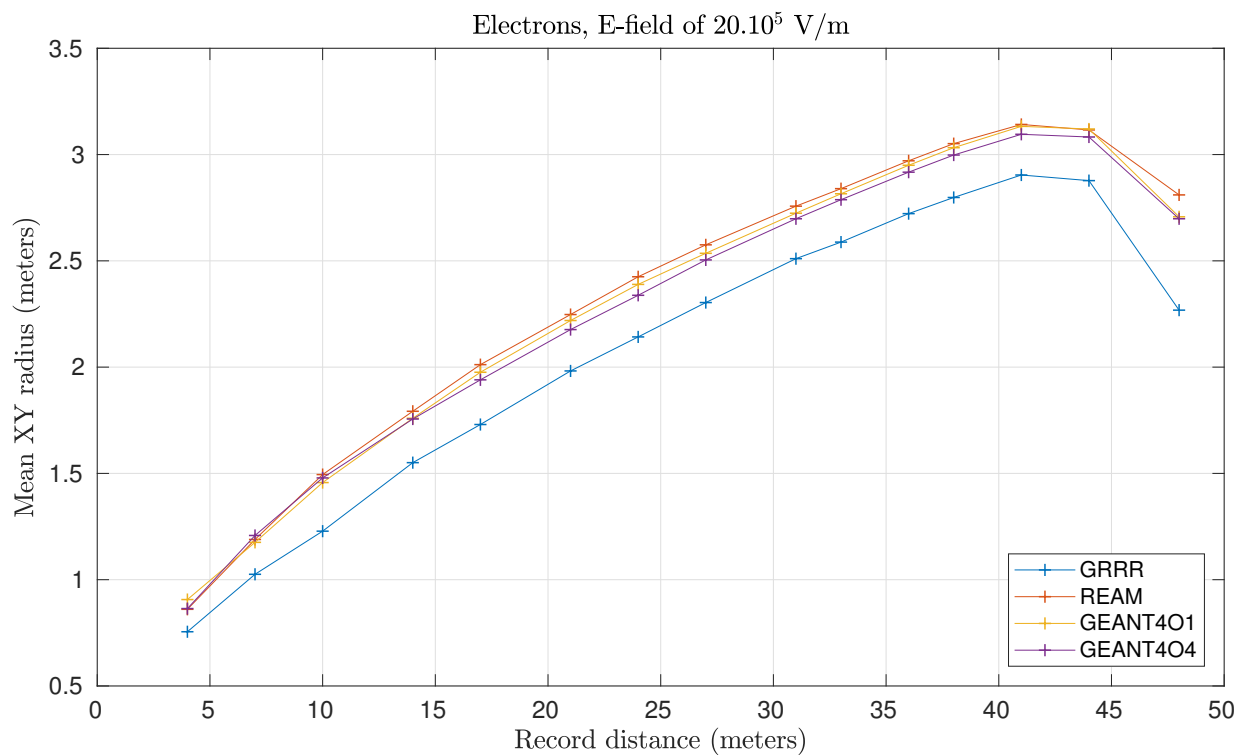
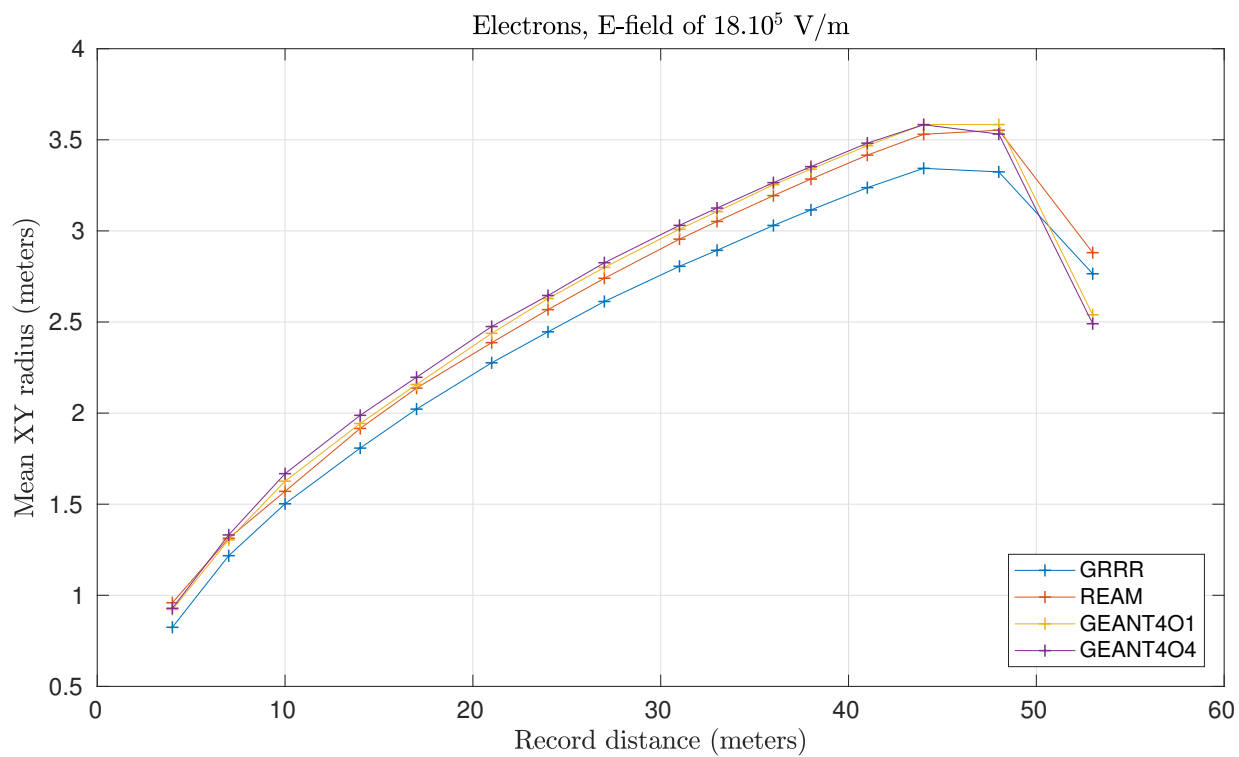
### 9.1 Code Comparison

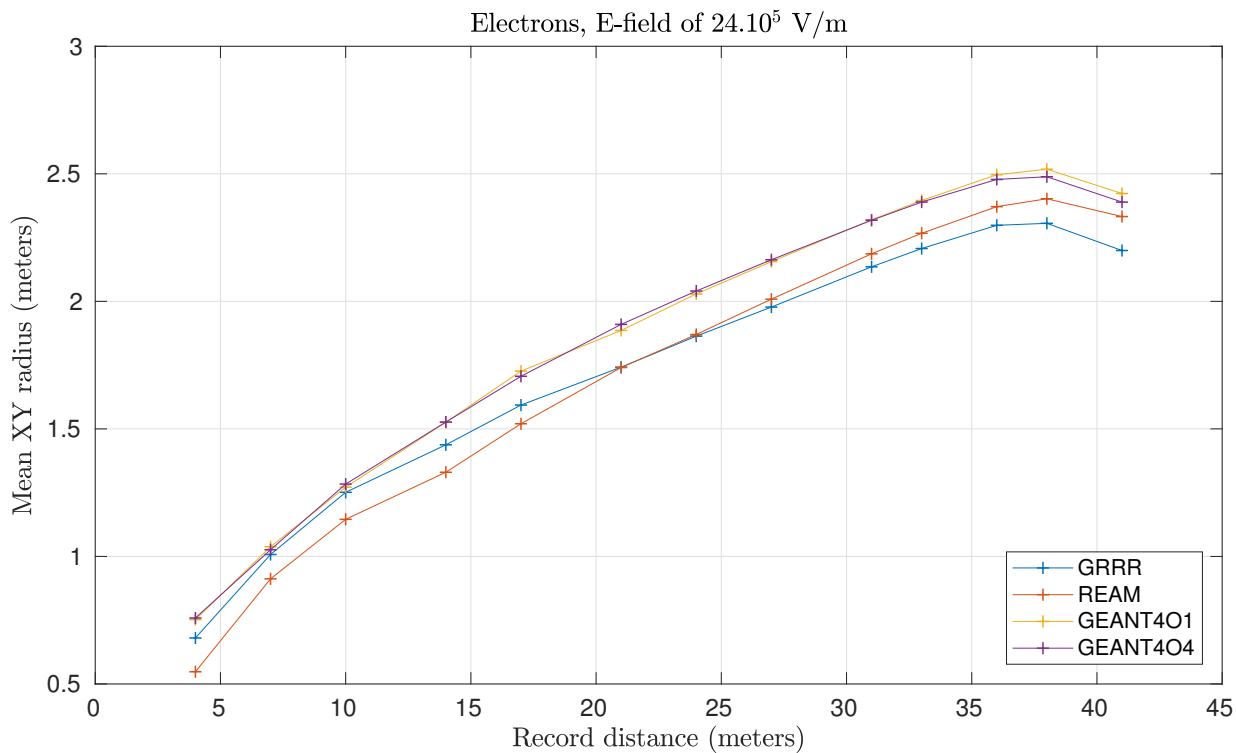
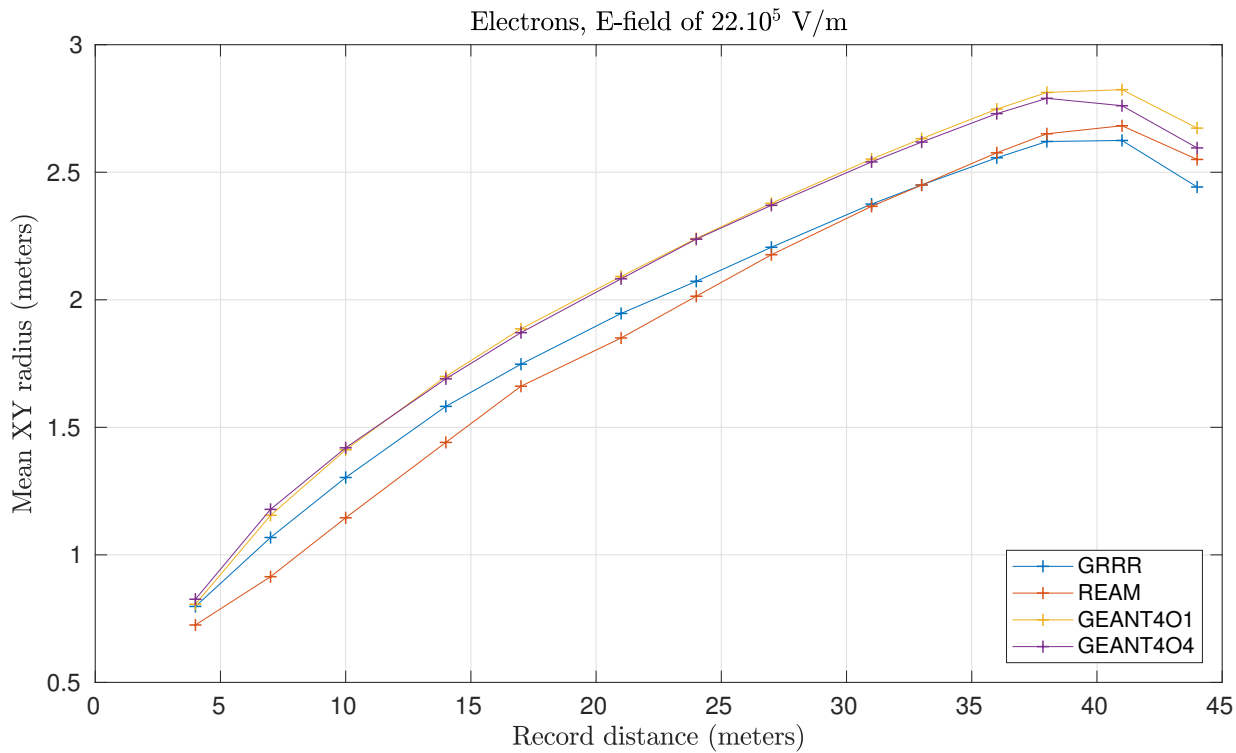


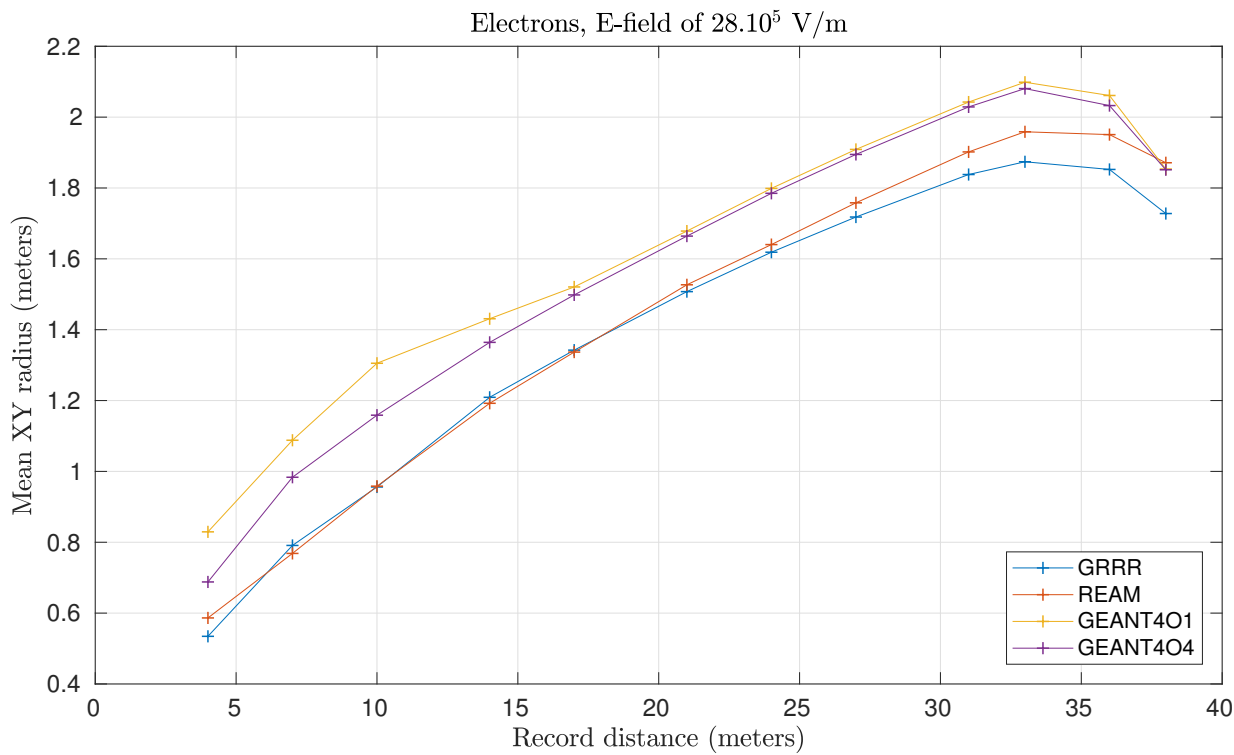
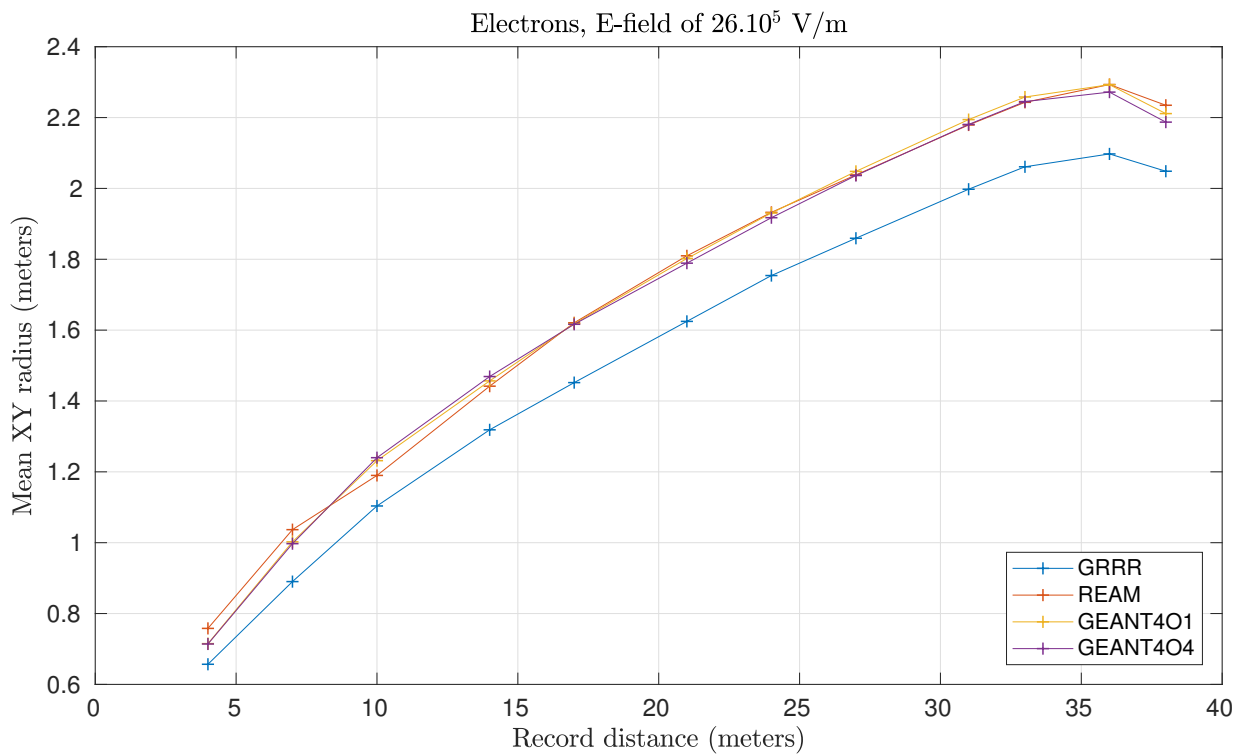




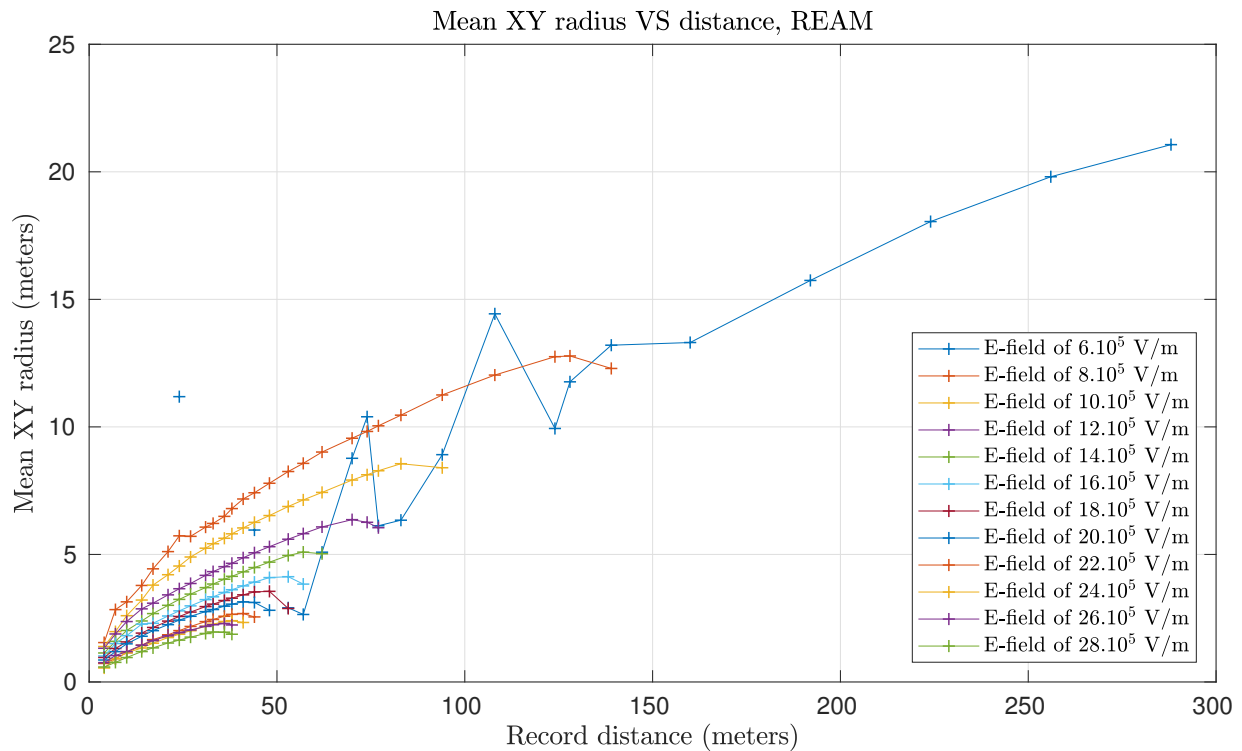
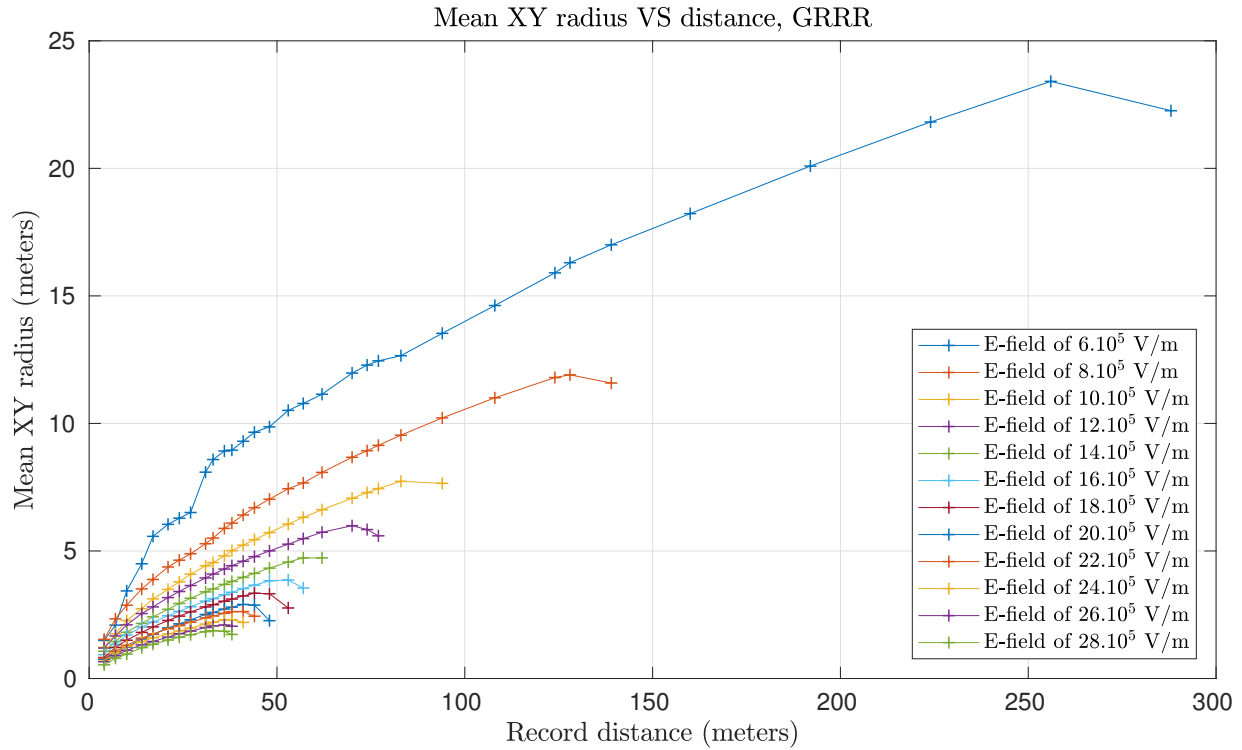


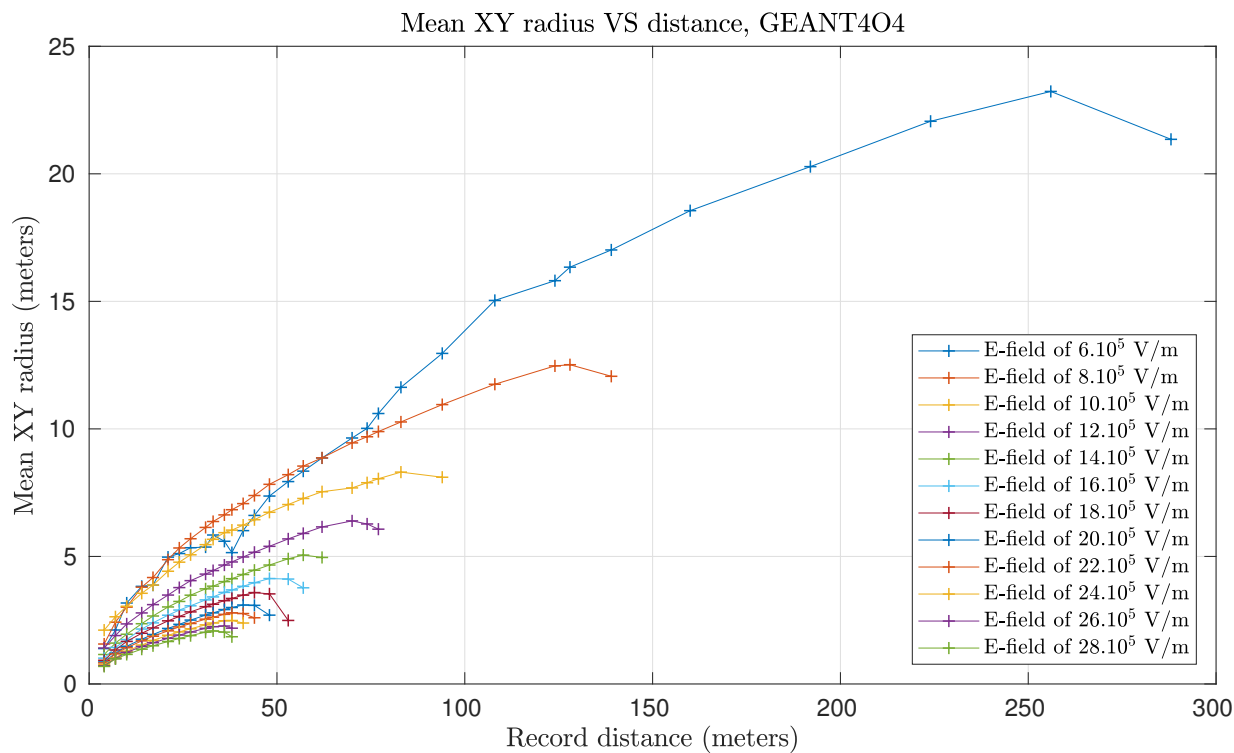
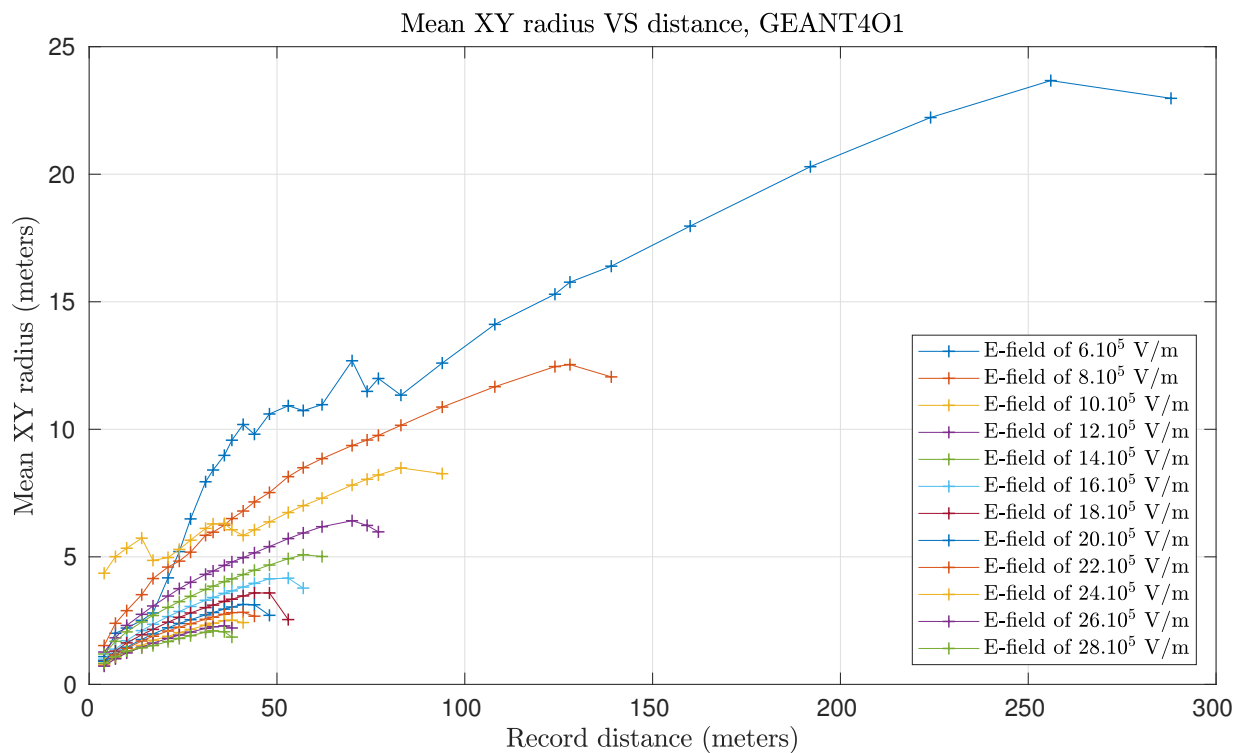






## 9.2 Code By Code



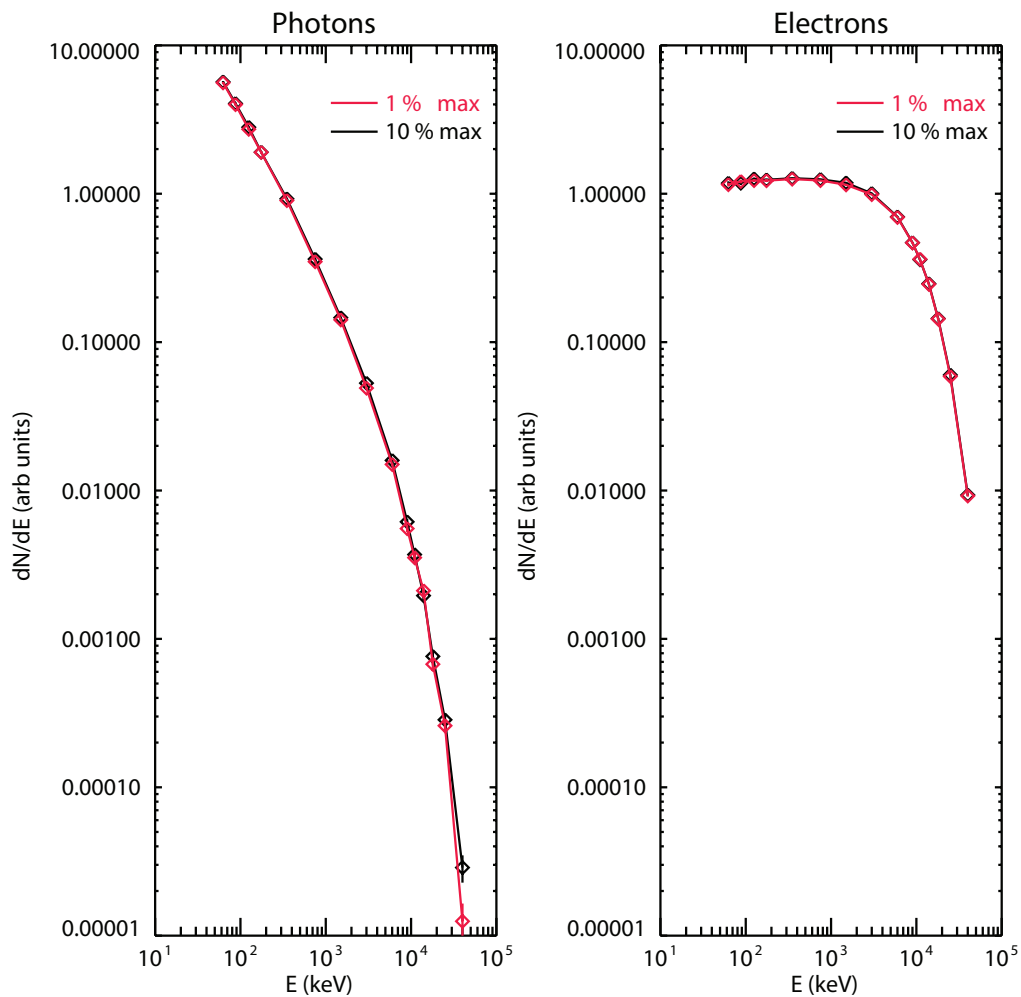


## 10 REAM Max Step

## 10.1 Spectra

## REAM, E-field of 1.2 MV/m, record at 77 m

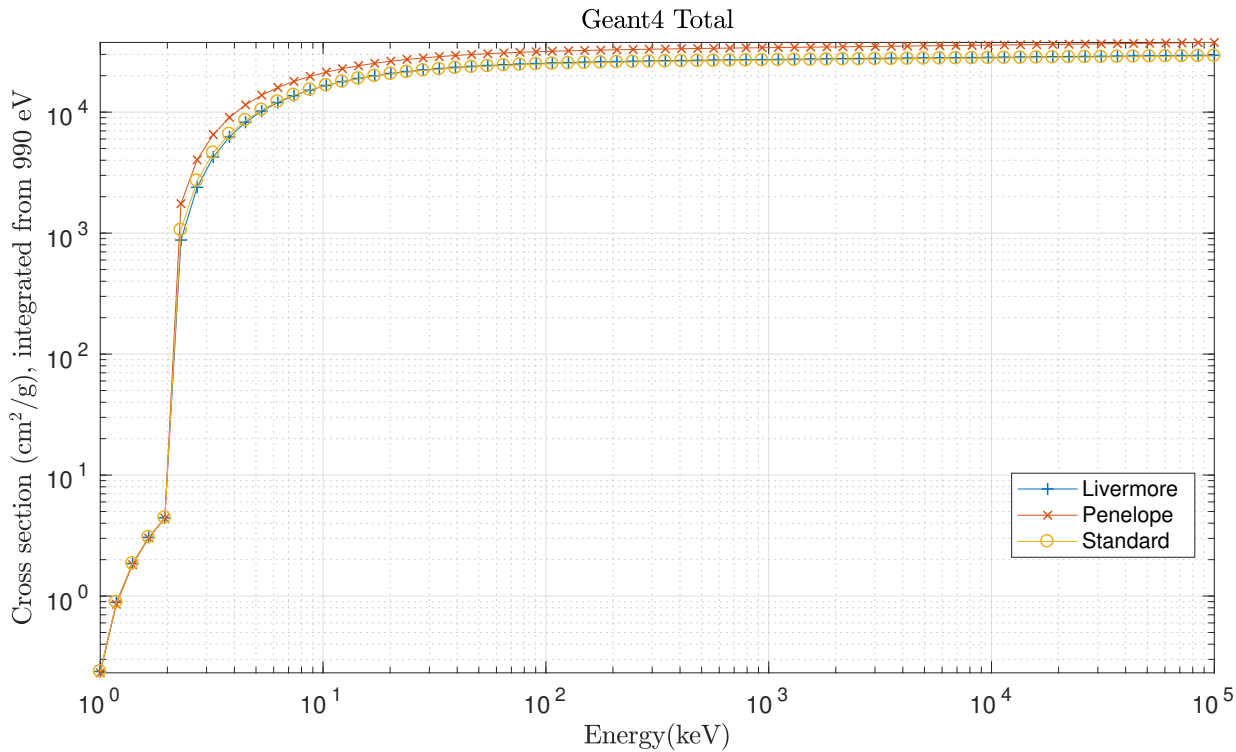
Test of maximum allowed change in energy during one time step



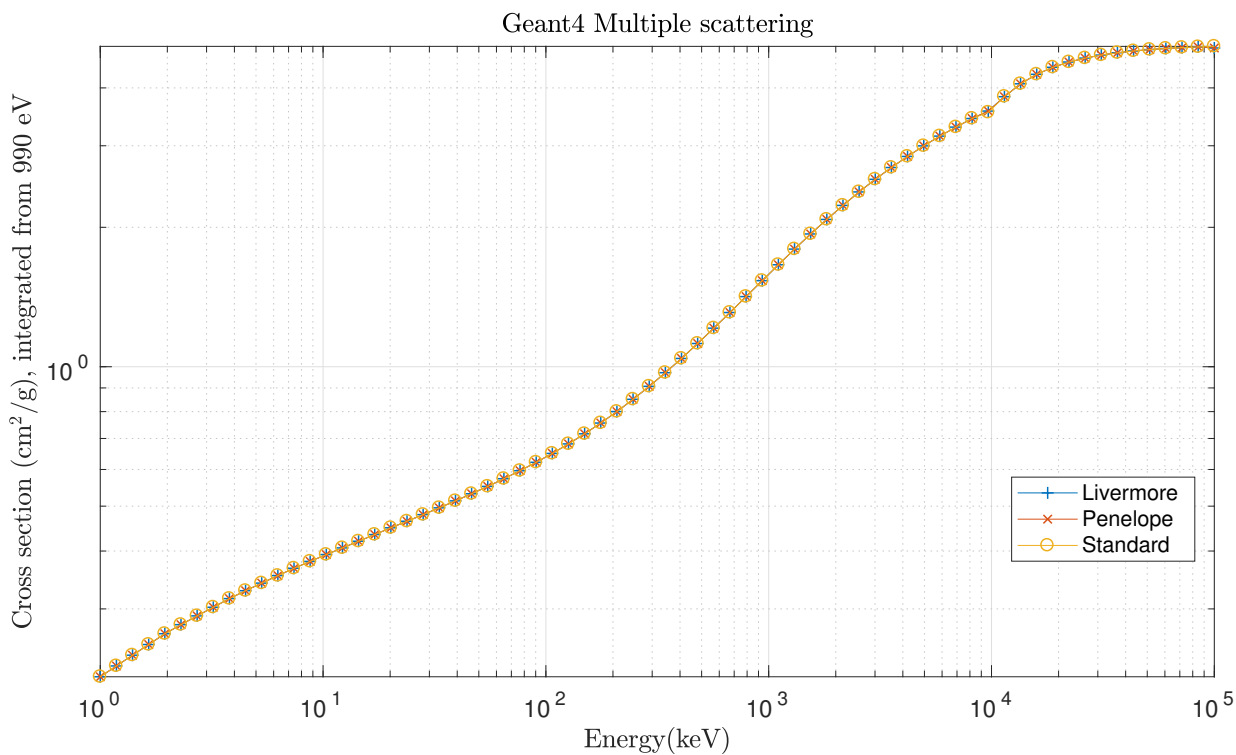


11 Geant4 Cross Sections Example Plots

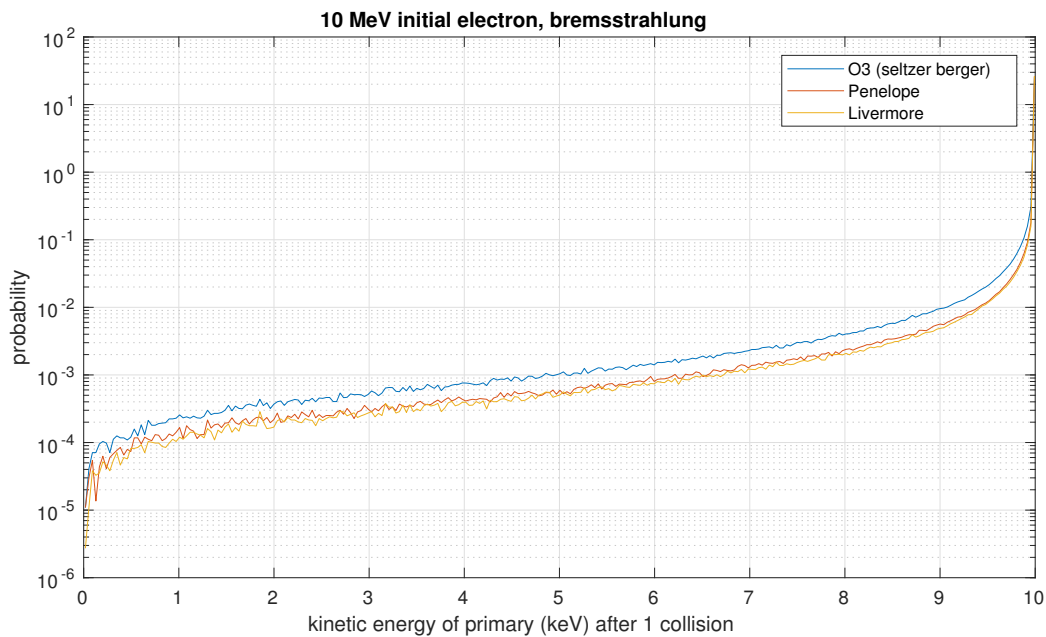
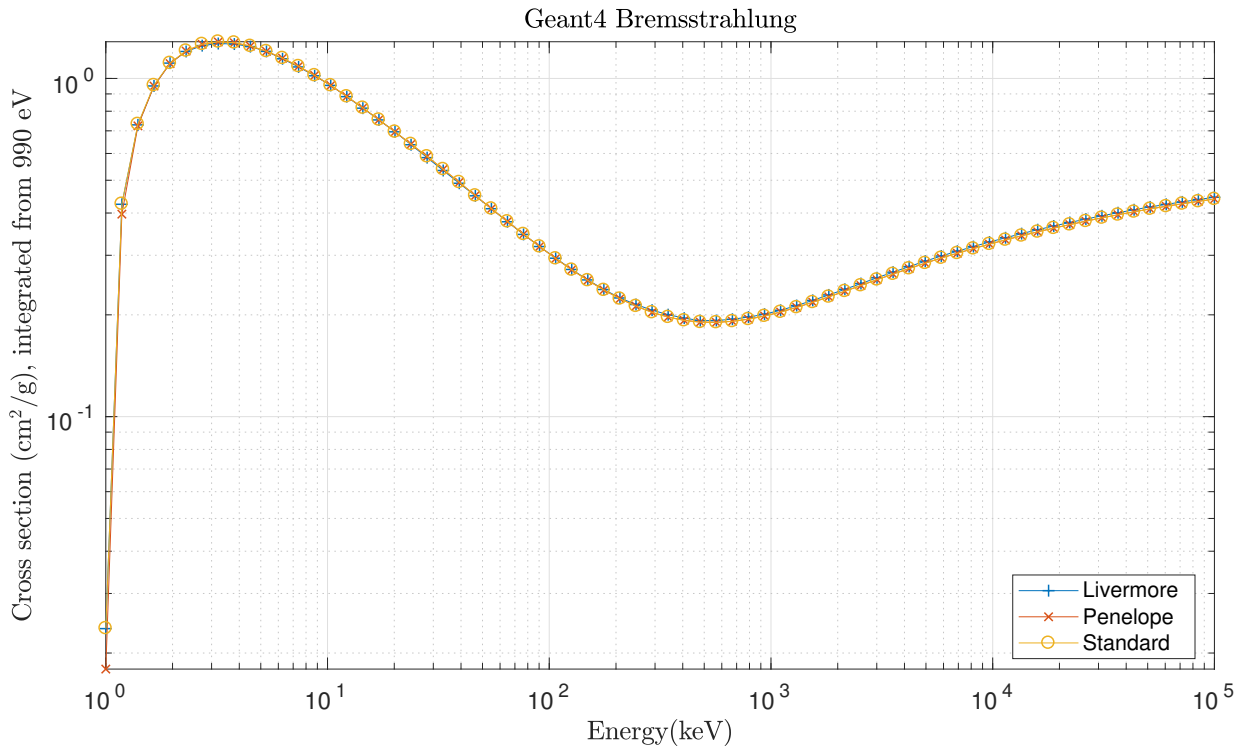
11.1 Total

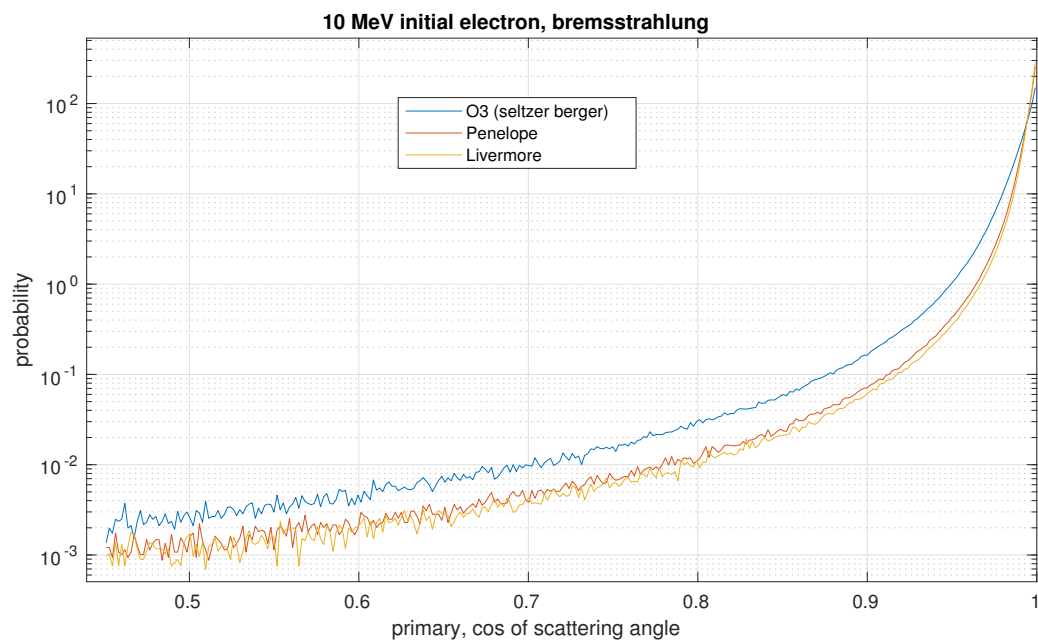
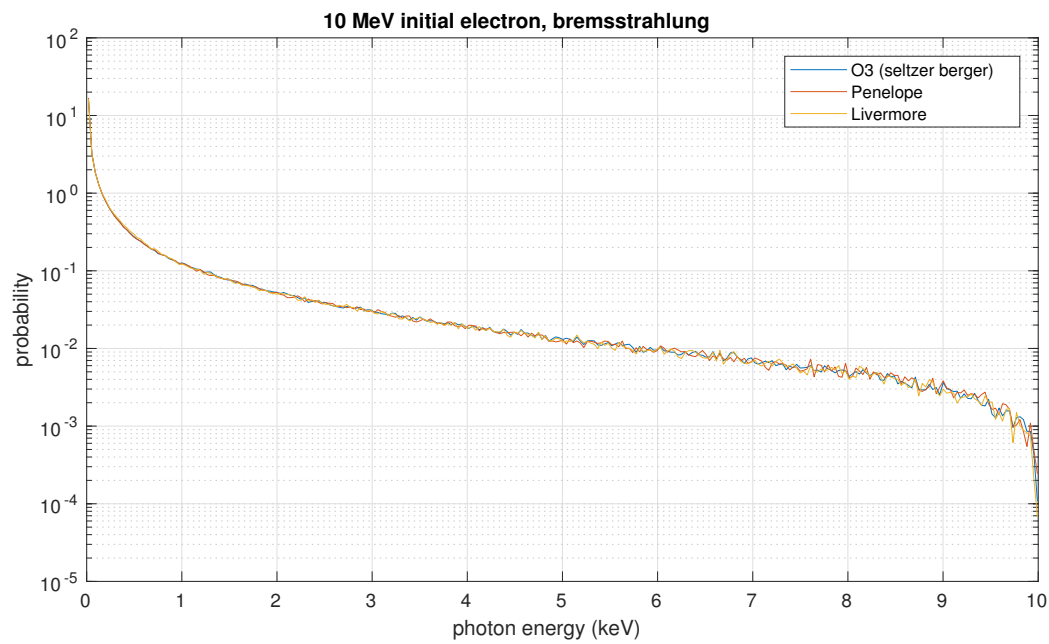


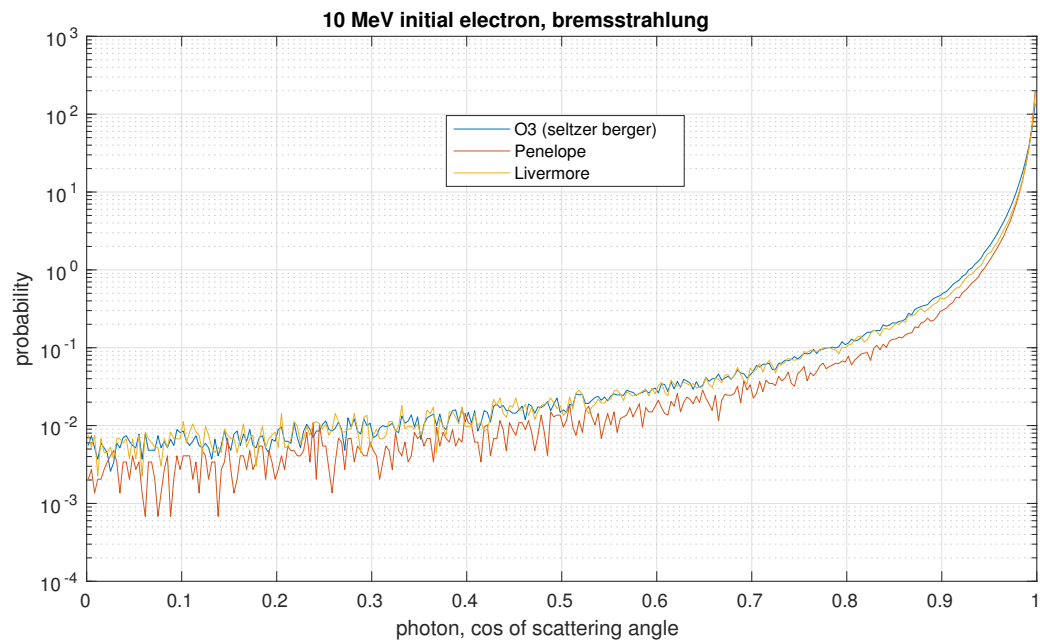
11.2 Multiple Scattering



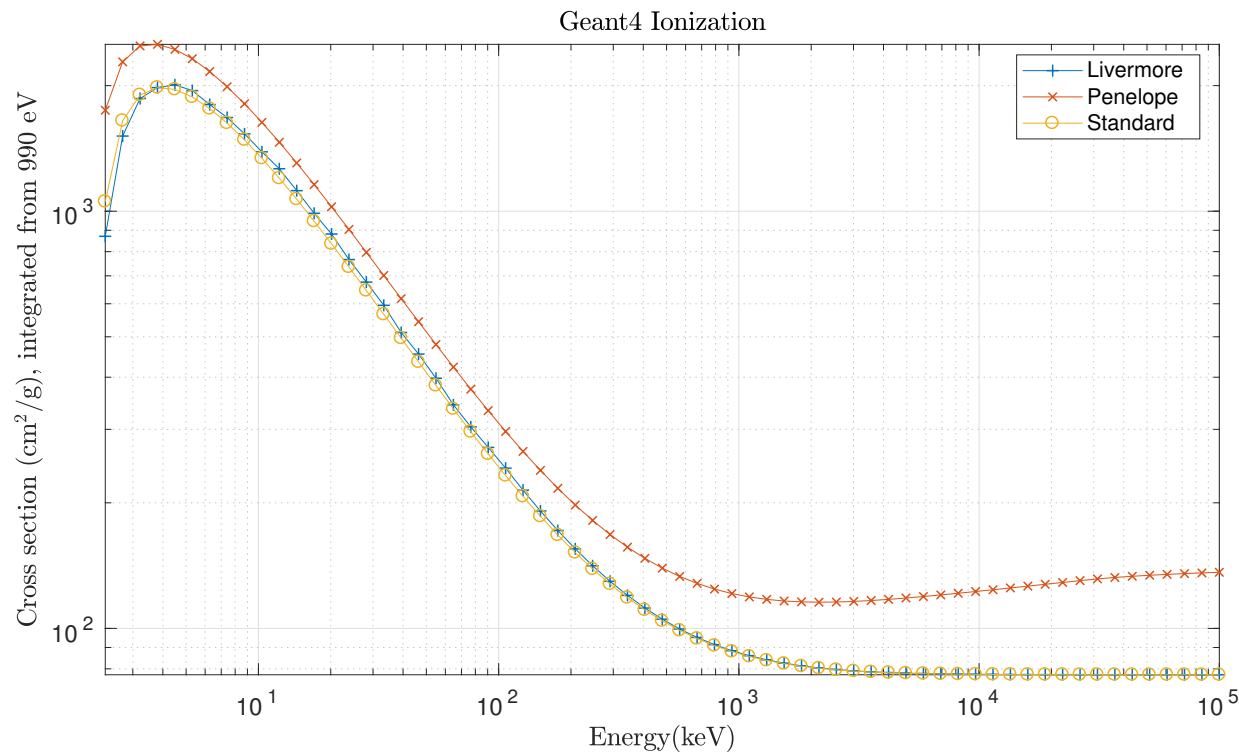
11.3 Bremsstrahlung

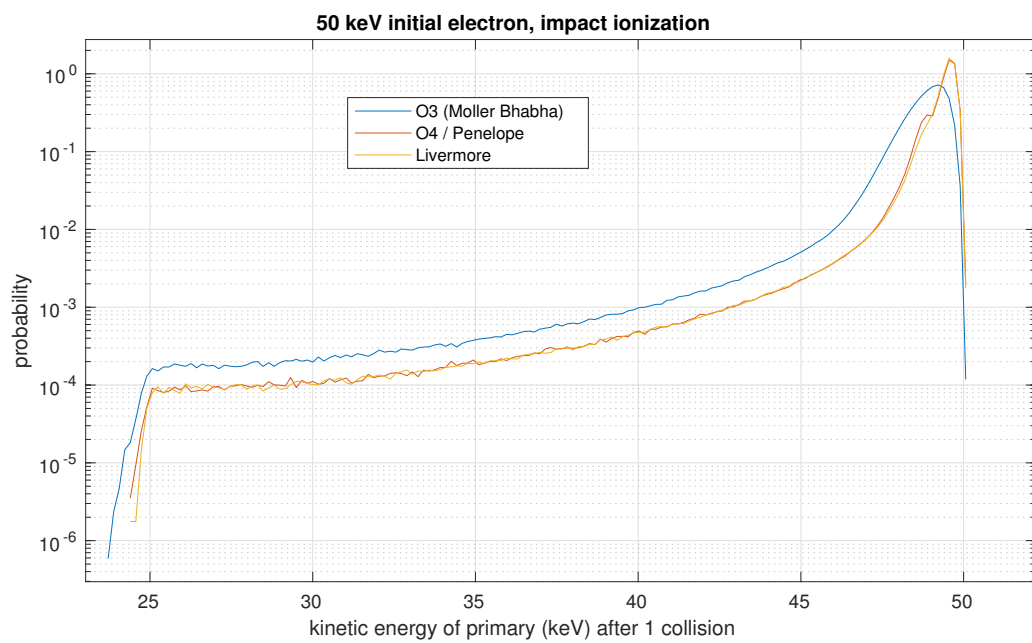
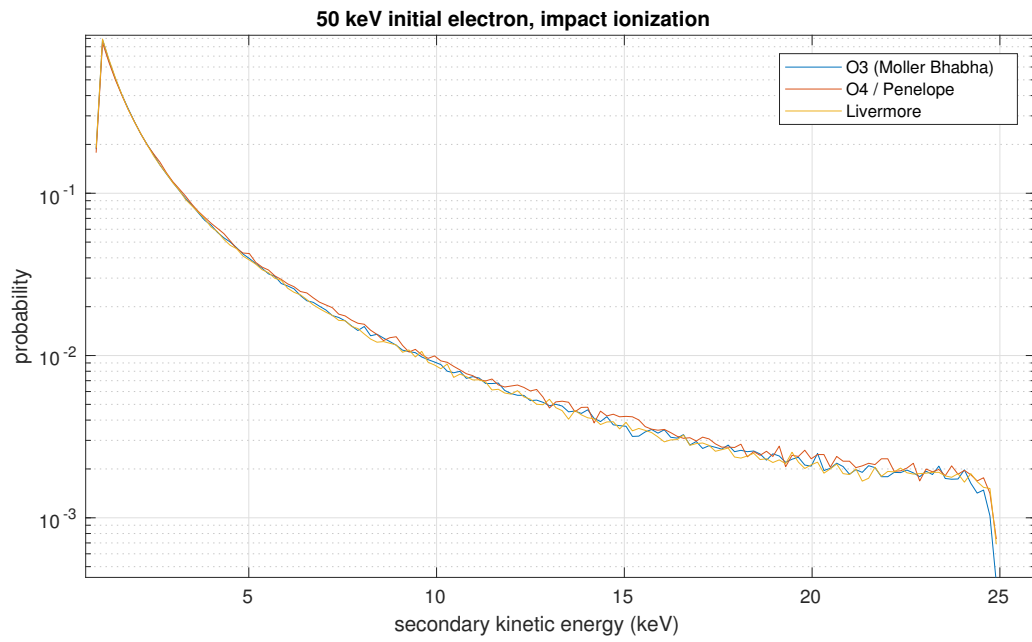


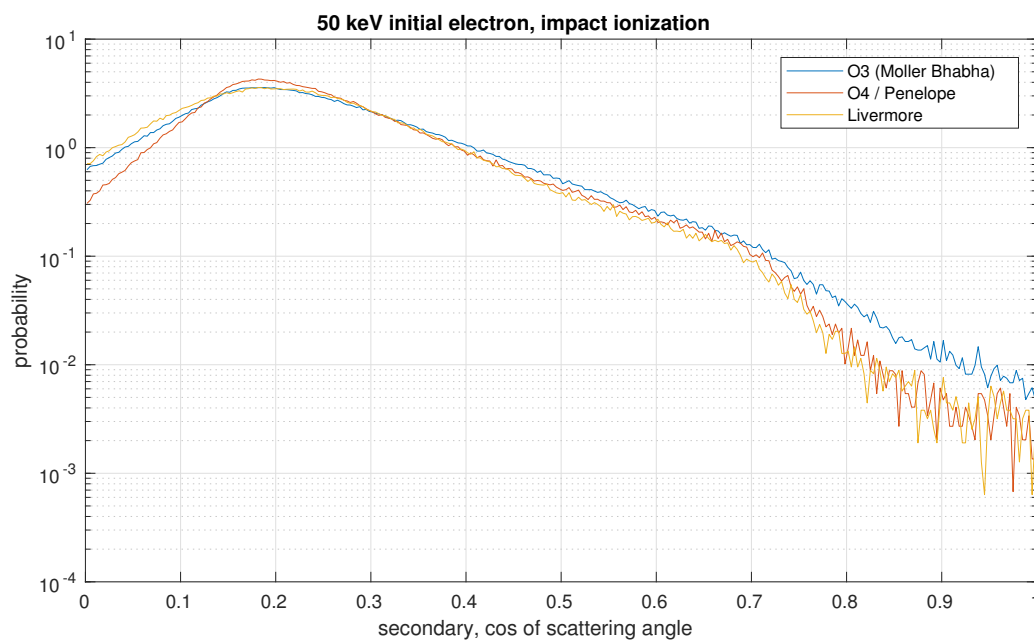
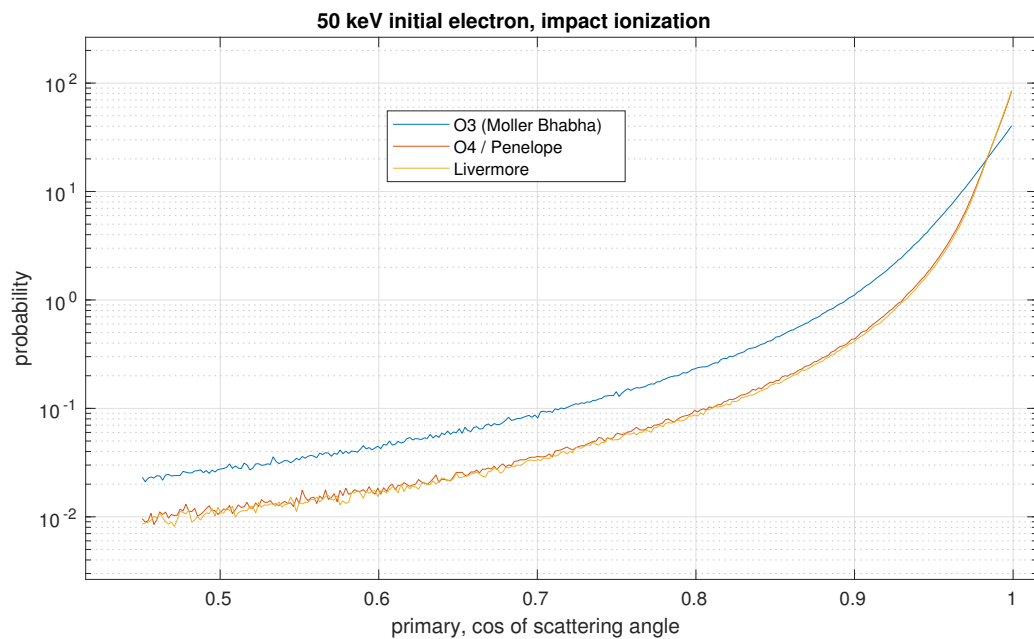




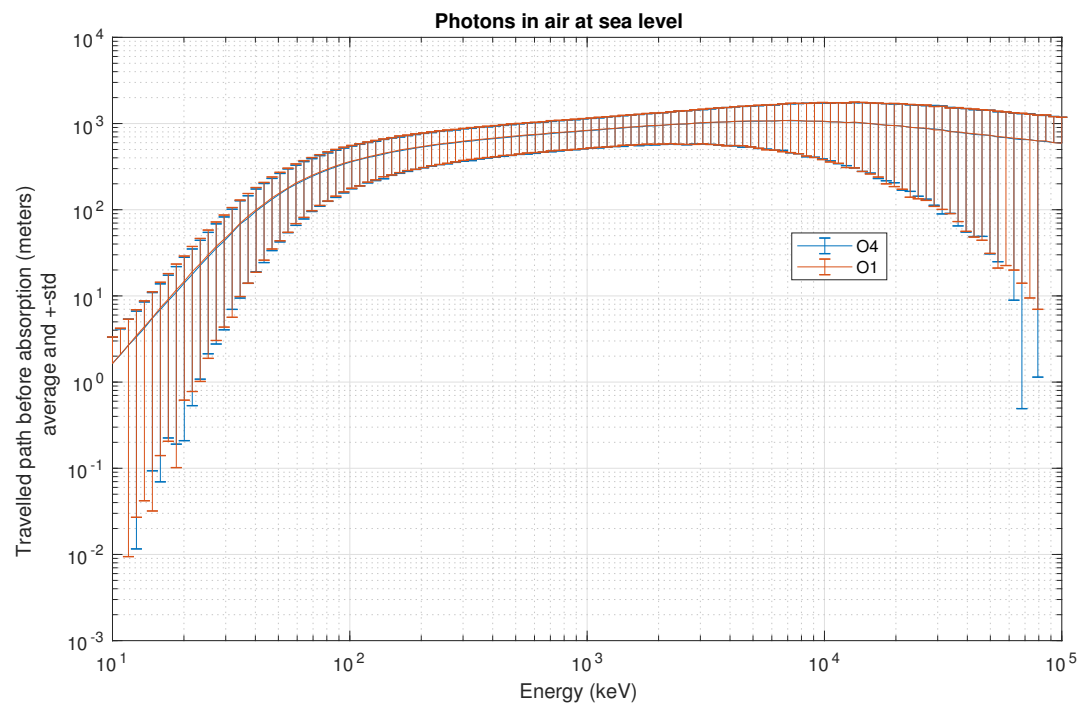
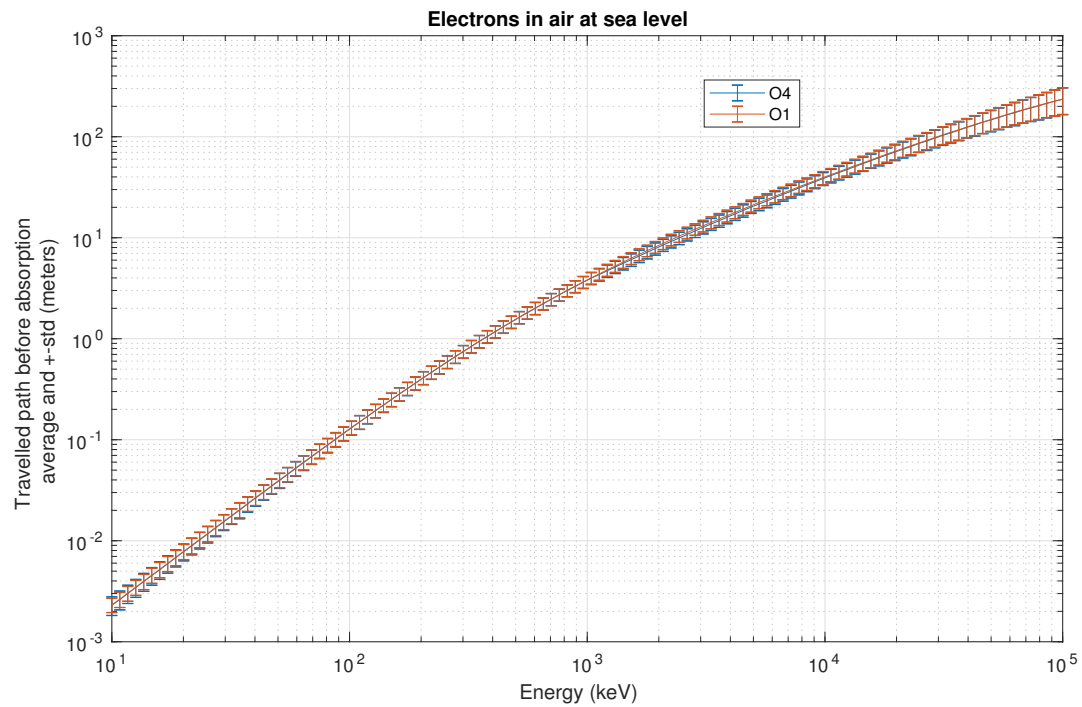
11.4 Impact Ionization





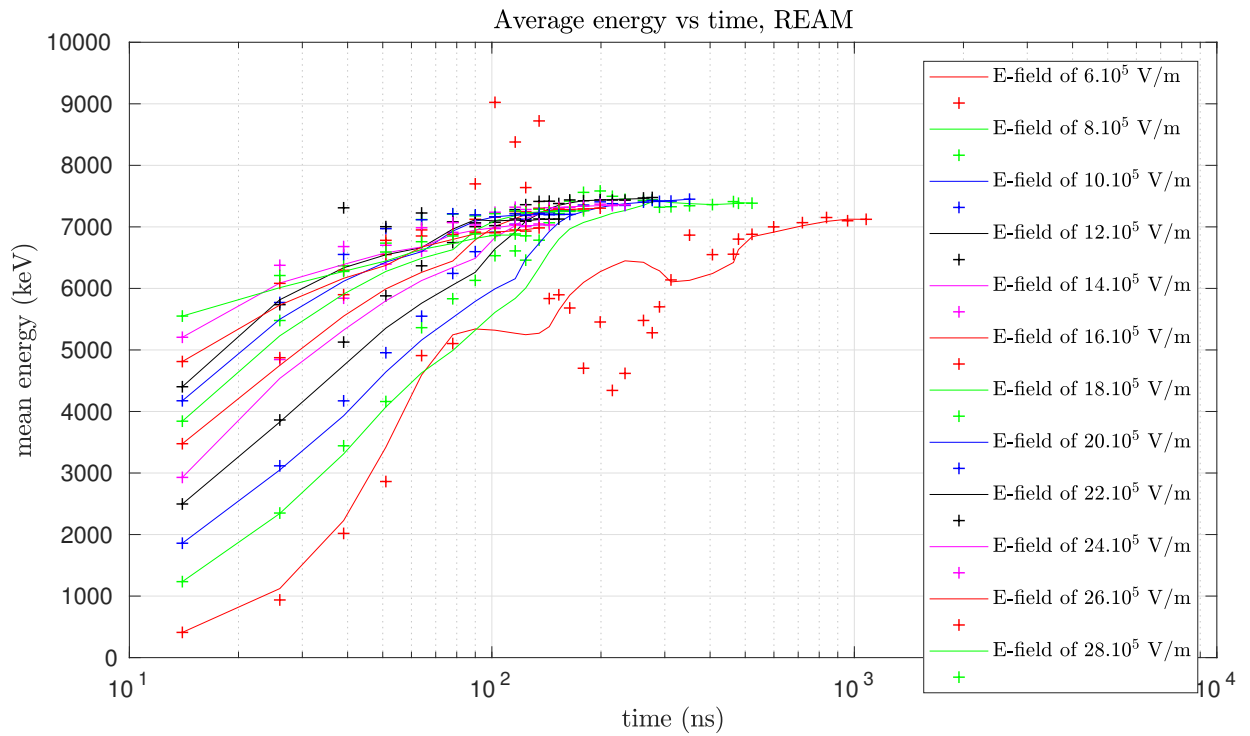
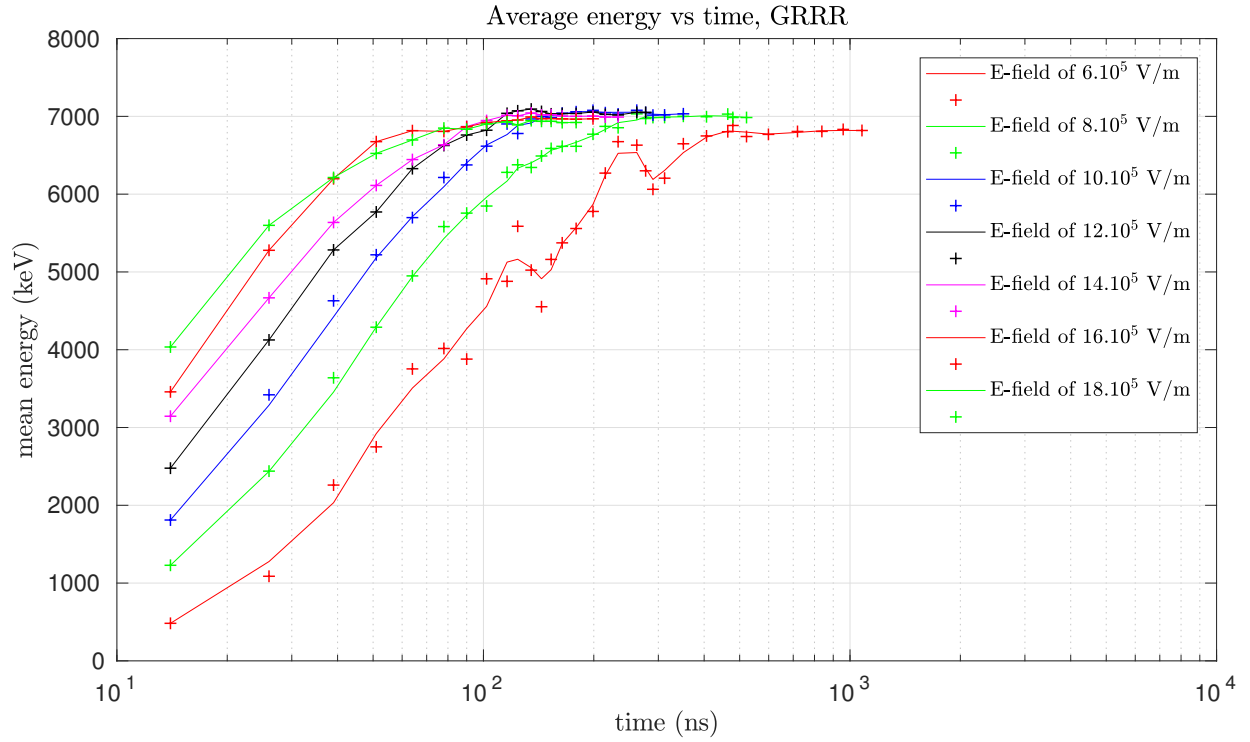


## 11.5 Travel Path Length

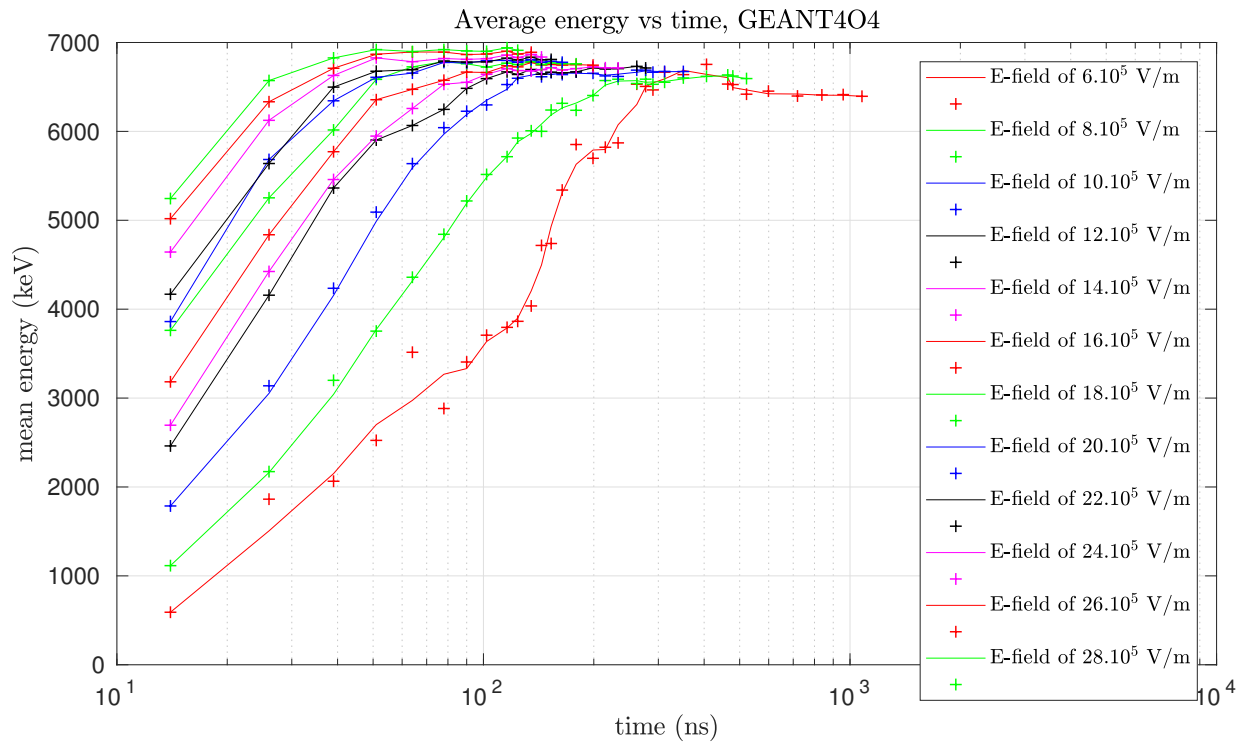
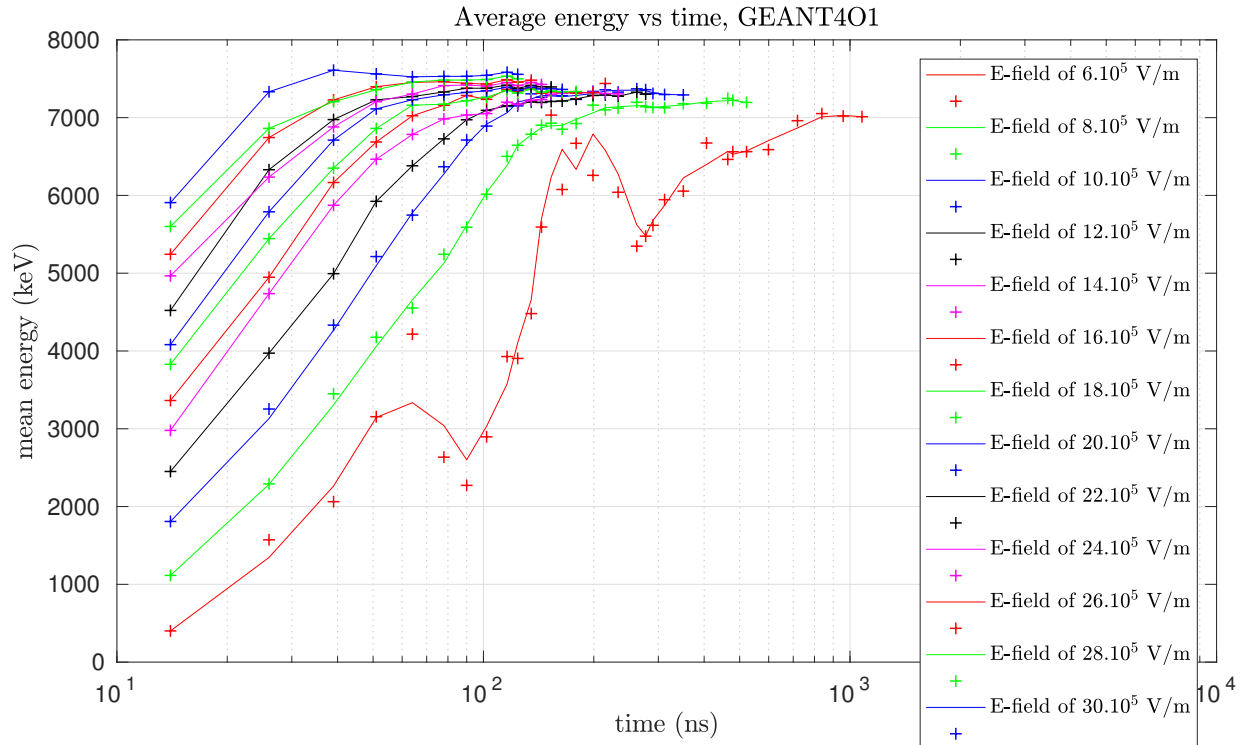


## 12 Time To Steady State

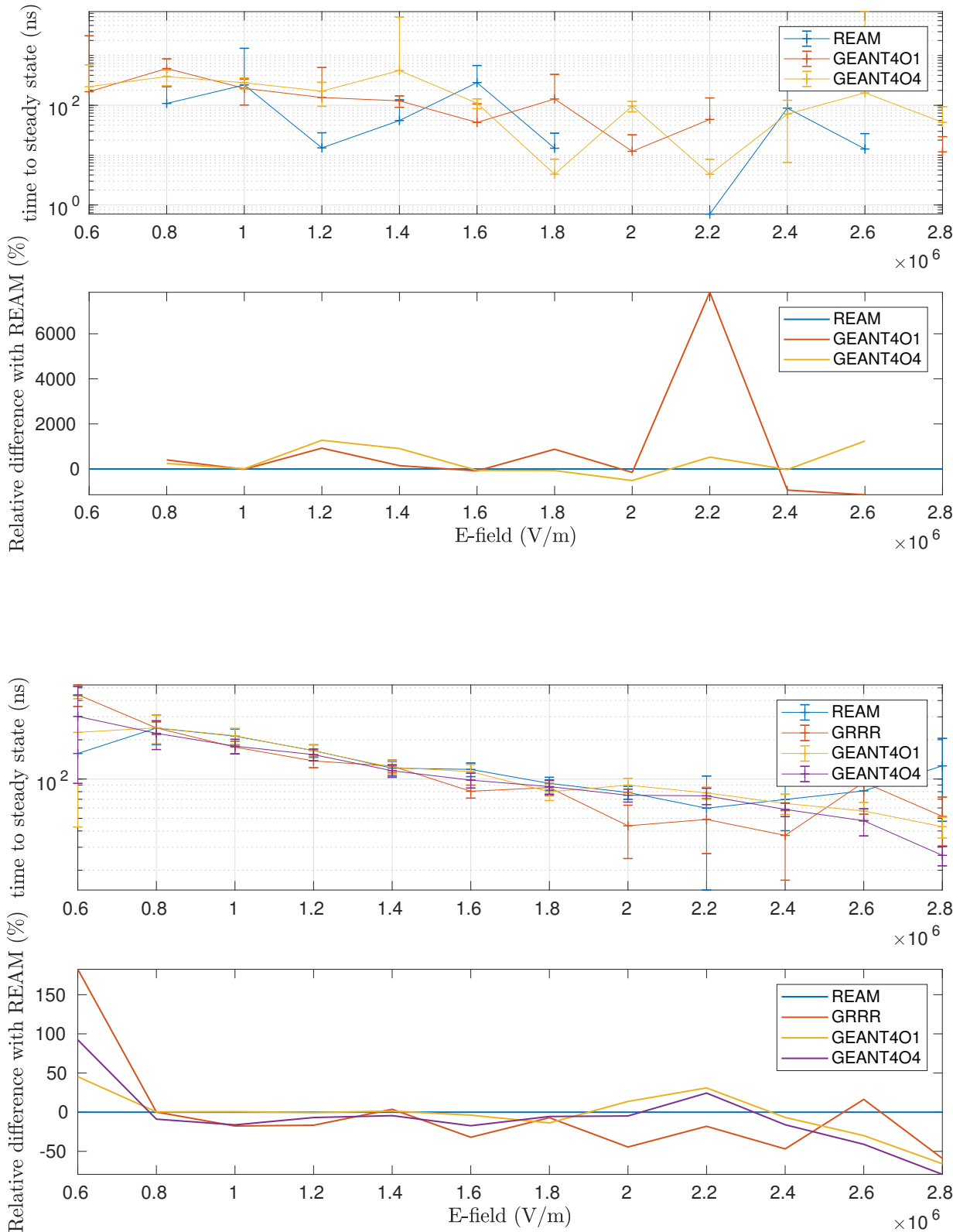
### 12.1 Average Electron Energy Evolution



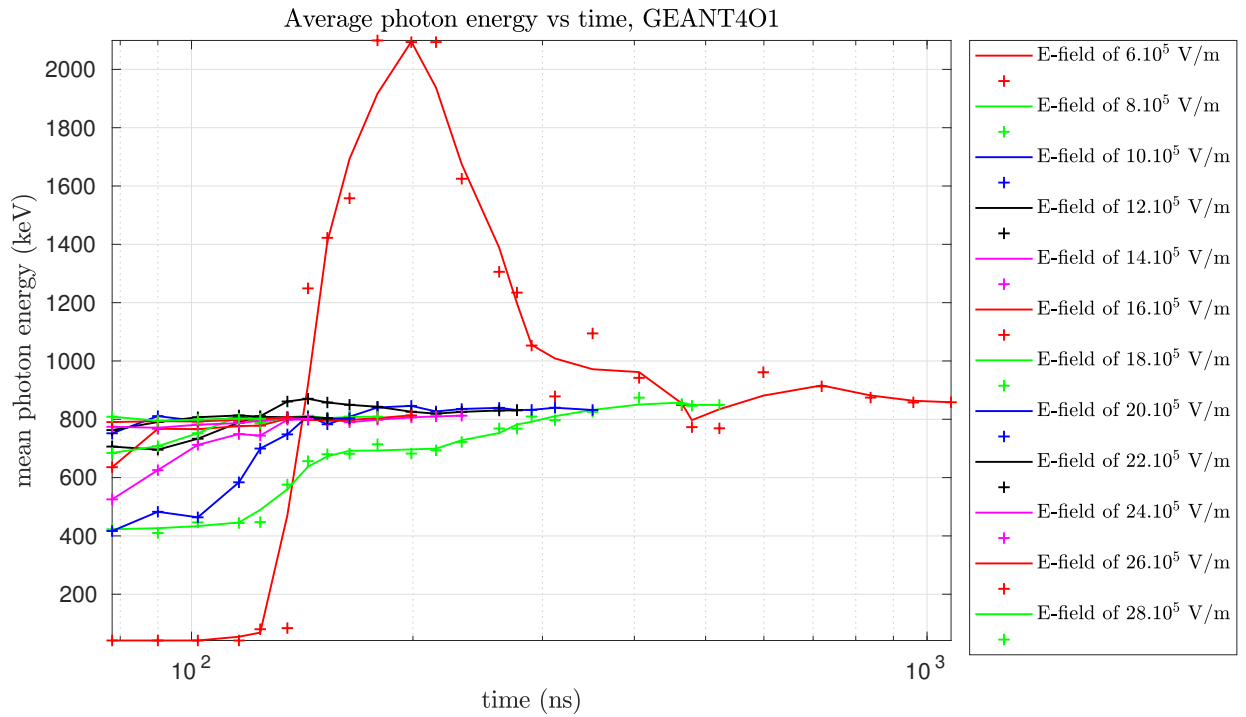
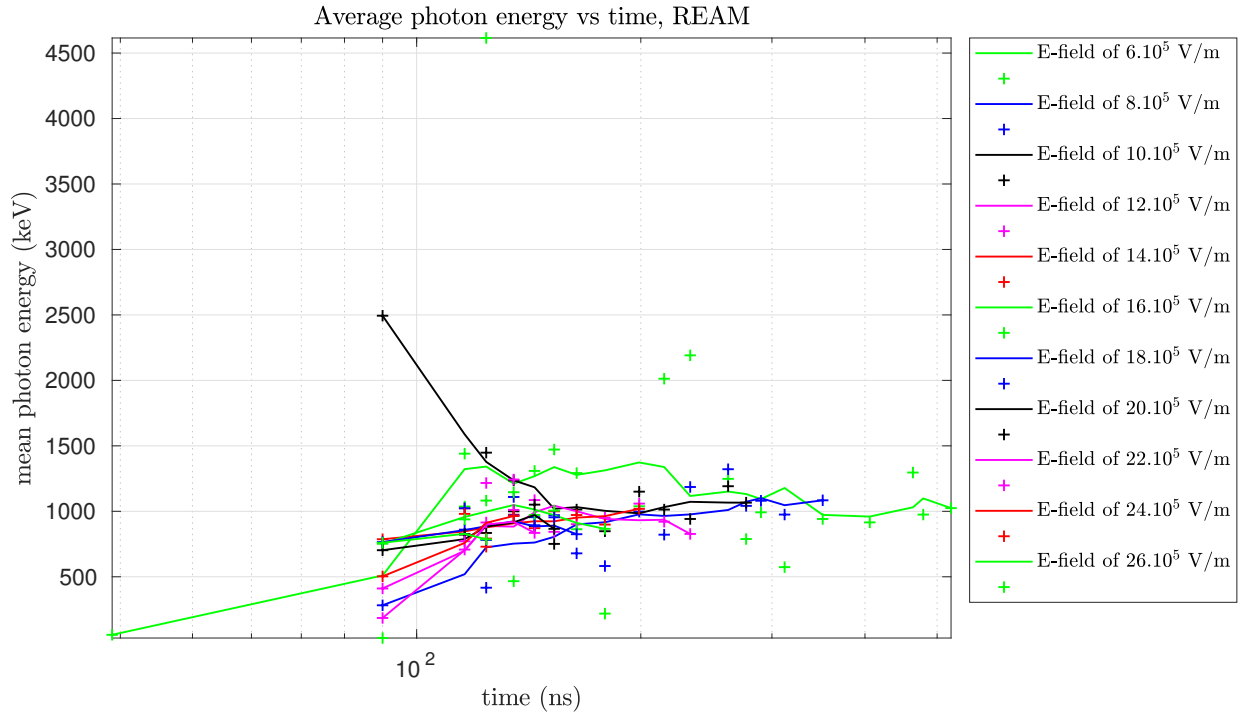


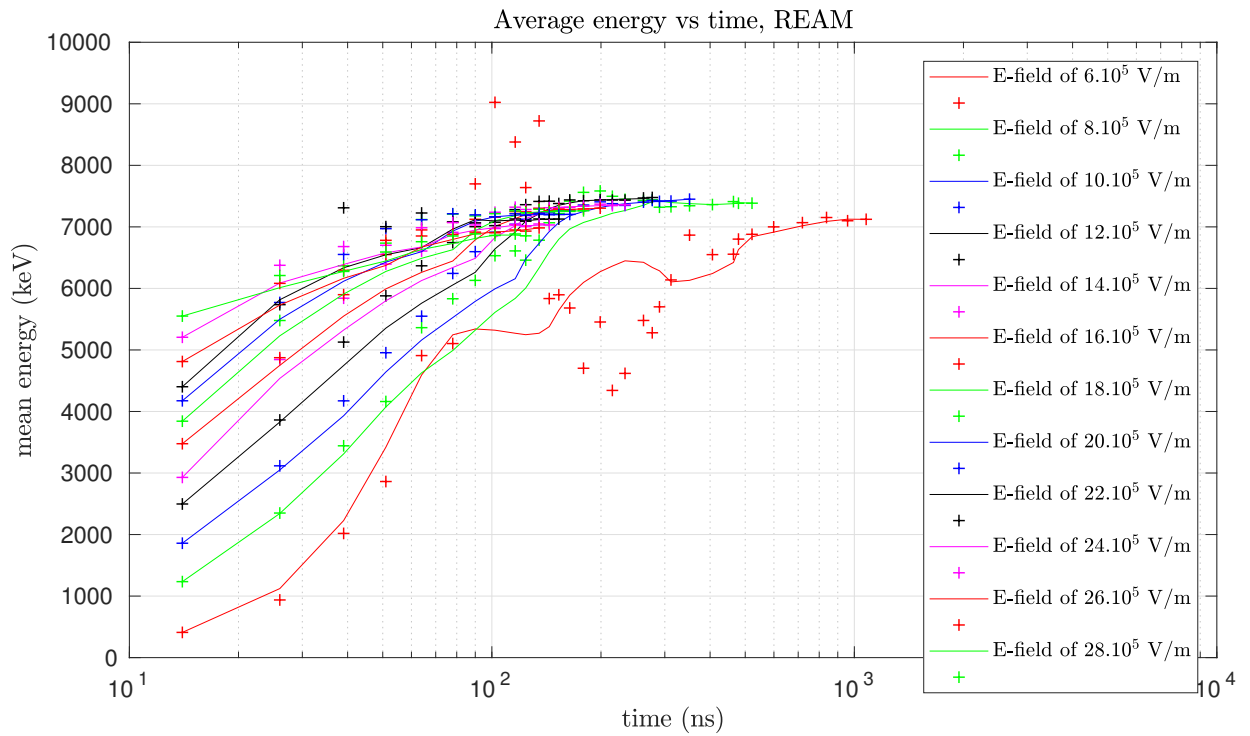
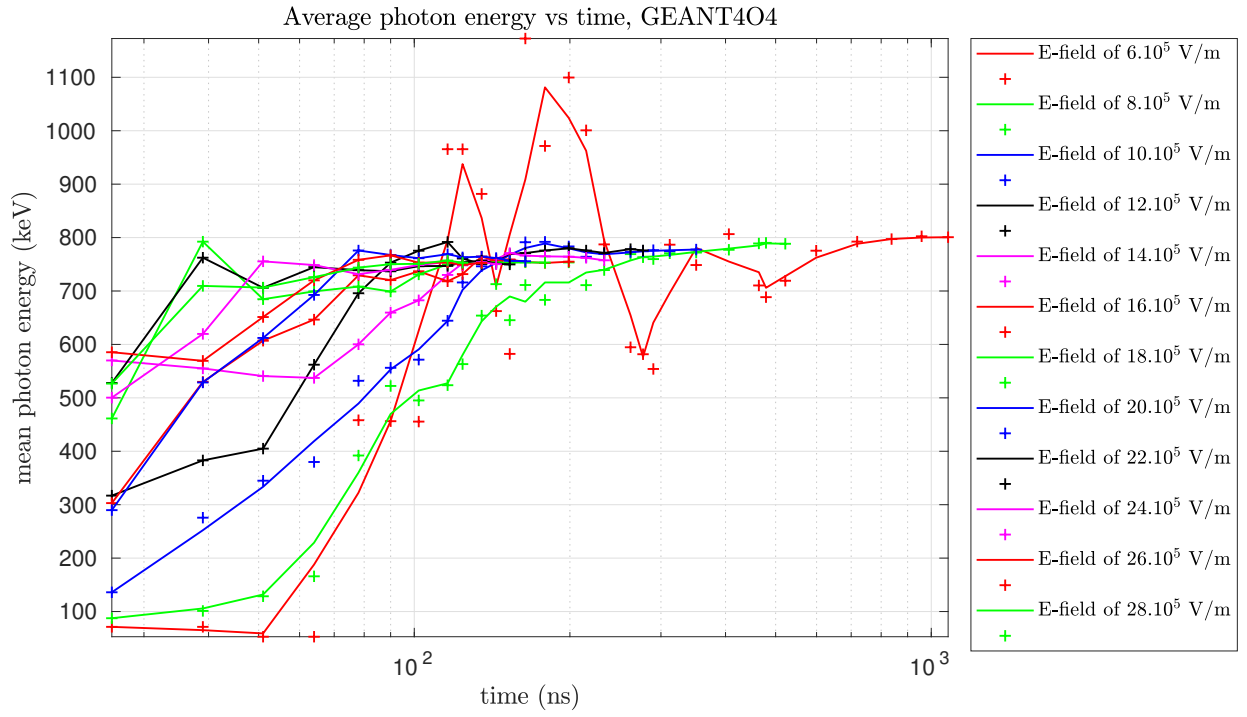


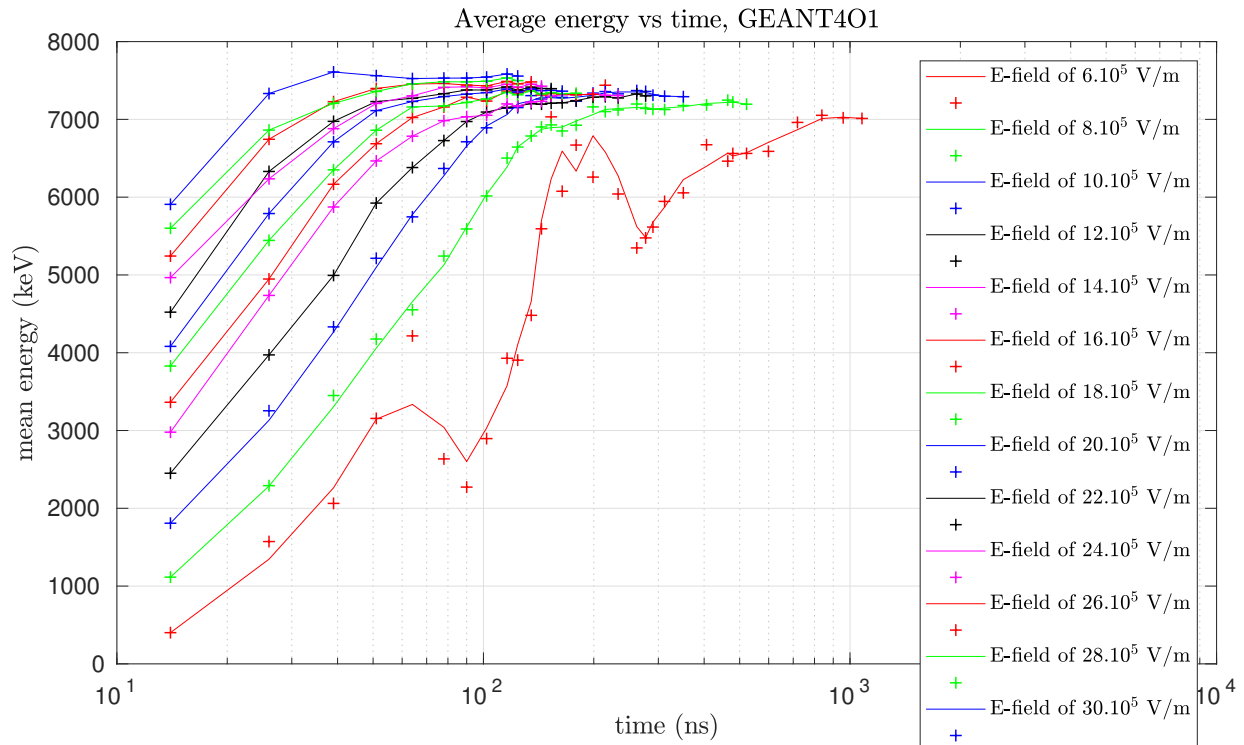
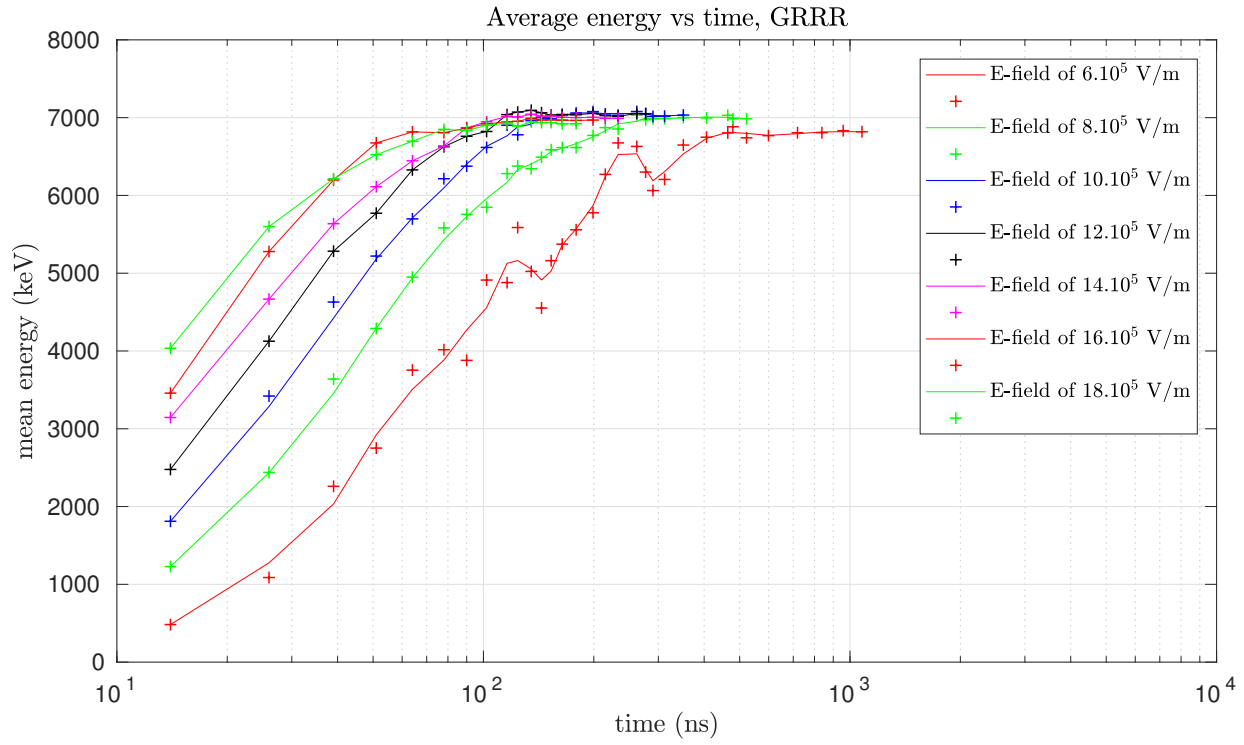
12.2 Comparison

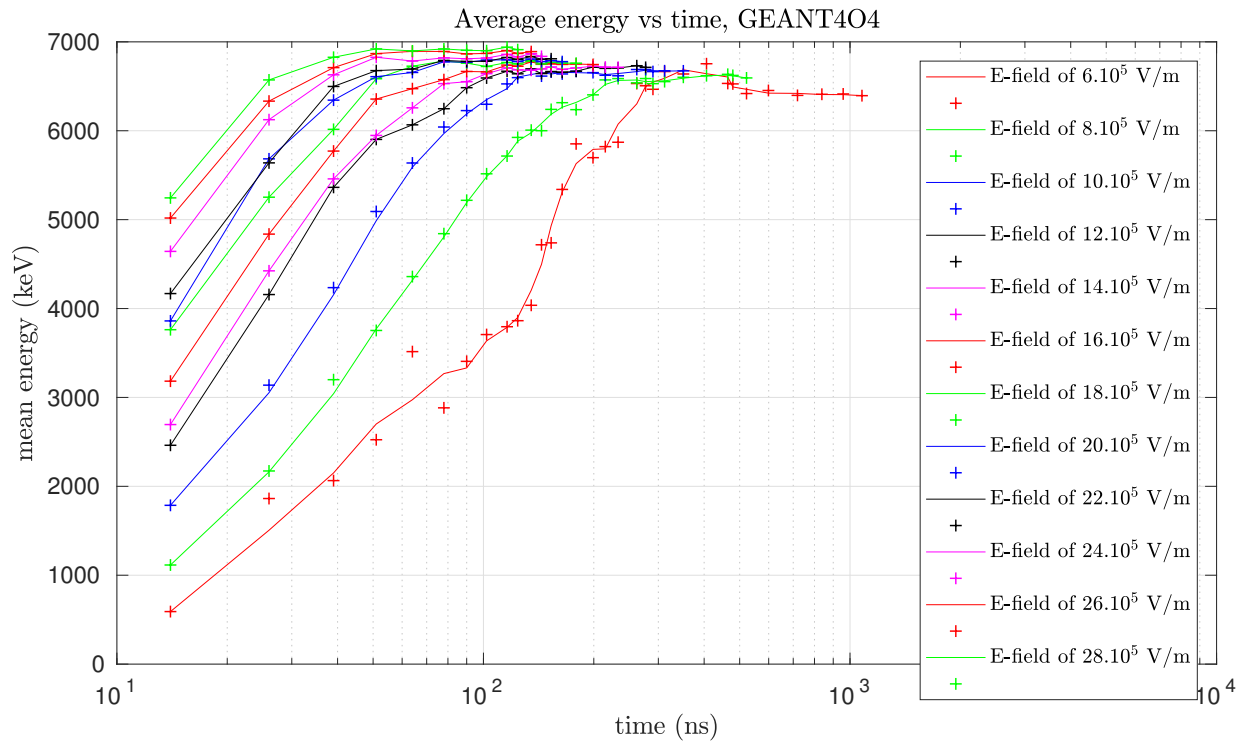


## 12.3 Average Photon Energy Evolution

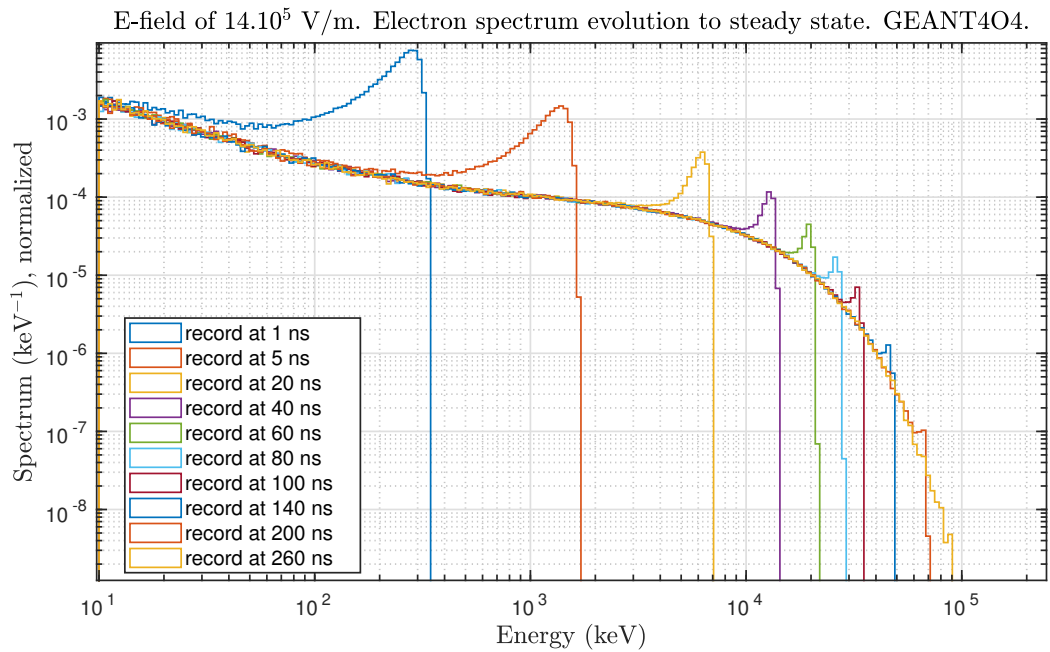


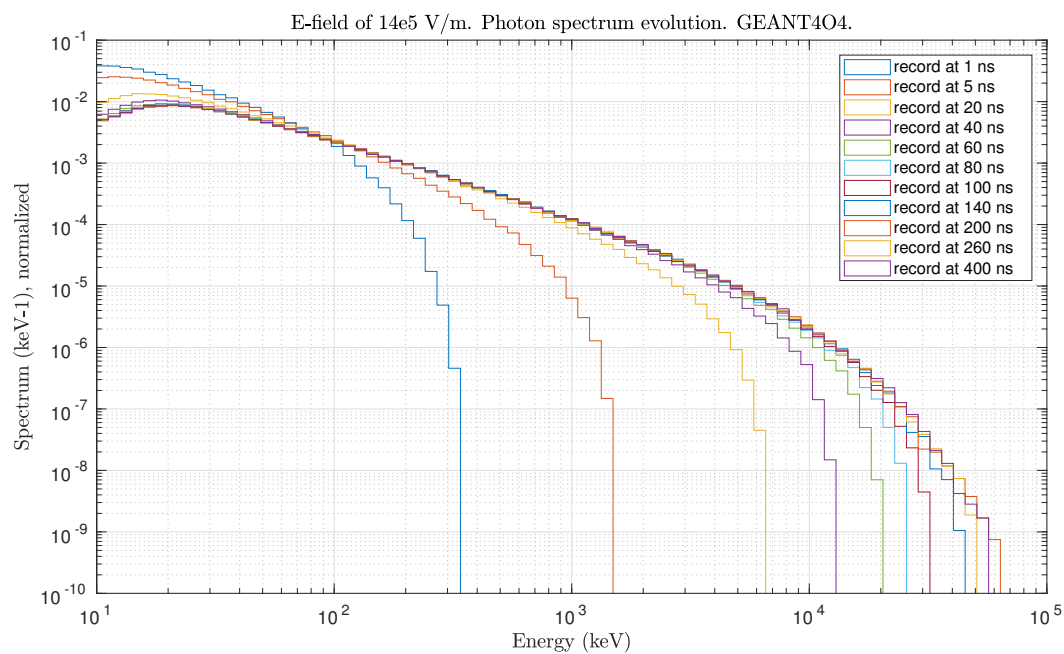






## 12.4 Spectrum Evolution To Steady State





## 13 Summary table of the processes and models used by the codes

		Cross sections / Models			
Particle	Process	Geant4 O1 [1]	Geant4 O4 [1]	REAM [5]	GRRR [4]
$e^-$ $e^+$	Ionisation Inelastic scattering	Analytical (Møller/Bhabha) ( <i>Messel and Crawford</i> , 1970) and [1]	0 to 1 MeV: Penelope [2] 1 MeV to 1 GeV: Idem O1	Bethe equation Møller scattering ( $e^-$ ) Bhabha scattering ( $e^+$ )	Møller scattering following ( <i>Lehtinen et al.</i> , 1999) ( <i>Dwyer</i> , 2007)
	Multiple scattering	Geant4 Urban model ( <i>Urban</i> , 2006) and [1]	Idem O1	N/A	N/A
	Coulomb (elastic) scattering (single scattering)	- neglected below 100 MeV - effects < 100 MeV included in the multiple scattering algorithm	Idem O1	Fully shielded Coulomb	Fully relativistic, shielded Coulomb from ( <i>Dwyer</i> , 2007) (similar to REAM, but another implementation)
	Bremsstrahlung	Seltzer Berger ( <i>Seltzer and Berger</i> , 1986)	Idem O1	Analytical ( <i>Dwyer</i> , 2007)	- Effective friction force - no hard collisions - data from ( <i>Berger et al.</i> , 2005) - neglected below 1 MeV
	Annihilation ( $e^+$ )	Analytical ( <i>Heitler</i> , 1984)	Idem O1	Analytical ( <i>Dwyer</i> , 2007)	N/A
$\gamma$	Photo-electric absorption	Parametrization of ( <i>Biggs and Lighthill</i> , 1988)	Livermore [3]	Based on data from NIST XCOM [6]	N/A
	Compton scattering	Klein-Nishina ( <i>Klein and Nishina</i> , 1929) and empirical formulas [1]	Monash University ( <i>Brown et al.</i> , 2014)	Klein-Nishina (similar to REAM, but another implementation)	N/A
	$e^- e^+$ Pair production	Tabulated and semi-empirical model ( <i>Berger and Hubbell</i> , 1987) ( <i>Baró et al.</i> , 1994)	Penelope [2]	Based on data from NIST XCOM [6]	N/A
	Rayleigh scattering	None	Livermore [3]	Analytical ( <i>Dwyer</i> , 2007)	N/A

Notes :

- [1] Concerning Geant4, more information about the cross sections and models can be found in the physics reference manual (*GEANT4 Collaboration et al.*, 2016): <http://geant4-userdoc.web.cern.ch/geant4-userdoc/UsersGuides/PhysicsReferenceManual/html/generalities/index.html>.
- [2] Penelope cross sections and models are described in (*Salvat et al.*, 2008)
- [3] Livermore cross sections and models are described in (*Cullen et al.*, 1997) and (*Perkins et al.*, 1991)
- [4] Concerning GRRR, more detailed information are given in the supplementary material of (*Luque*, 2014).
- [5] Concerning REAM, more detailed information are given in (*Dwyer*, 2007).
- [6] The NIST XCOM database is available here: <https://physics.nist.gov/PhysRefData/Xcom/Text/intro.html>



## References of the table (previous page)

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#### 14 TGF photon probability to escape the atmosphere

Assumptions:

- Photon spectrum:  $\propto \frac{1}{E} \exp\left(\frac{-E}{7.3 \text{ MeV}}\right)$
- Source altitude: 12 km
- 5 – Angle beaming: Isotropic with 45 degrees opening angle around zenith.
- Record at 500 km (atmosphere negligible above  $\sim 120$  km for photons)

